A LABORATORY MANUAL OF ALTERNATING CURRENTS

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PREFACE.

In composing a set of notes intended for laboratory use the writer has to consider two possible methods of treating the subject. One is to describe explicitly the different tests to be carried out, giving diagrams of connections, meters to be used and readings to be taken, and even giving the form of the log for keeping the laboratory data. Practically nothing is said regarding the theory involved in the test, the reasons for taking the various measurements, necessity of holding certain quantities constant and allowing others to vary, etc. This method of treatment makes the performance of the laboratory work extremely simple for the student as well as for the instructor. It is probably for this reason that most laboratory texts are written in this style. Such a method of presenting the subject to the student, however, necessitates very little thinking on his part. The connection of the apparatus and taking of readings are reduced to a more or less mechanical operation, and even the keen student does not get from his laboratory work nearly as much material for study as he should.

The second method of handling the subject matter to be investigated by experiment consists in a careful analysis of test to be performed, of the different variables involved, their relations to one another, errors likely to be introduced, etc. As for the actual performance of the test, the student should, in so far as is feasible, be made to work out for himself the scheme of connections, meters to be used, readings to be taken, arrangement of data on the record sheet, etc. That such a course of analysis of the test to be made is the proper preparation for the student to have before performing his laboratory work is evident when it is remembered that the practicing engineer must do just exactly this before carrying out any commercial test.

It is, of course, appreciated that the lack of specific directions in a laboratory text means more careful preparation by the student and also more work for the instructor, but as a laboratory course should be primarily designed to teach the student

methods of analysis, and to emphasize the theory presented in lecture courses, rather than to facilitate the perfunctory performance of a set of experiments, this method is thought to be the proper one and so is used in the following text.

In this book, intended for a laboratory manual, it would evidently be out of place to try to give a complete mathematical analysis of all alternating-current phenomena. The various lecture courses, which this laboratory course is supposed to parallel, will take up the general theory of alternating current circuits and machinery, so that the author has attempted to give here only those elements of the theory which apply directly to some phenomenon to be experimentally investigated.

The analysis of the action of alternating current meters has been given more in detail than is given elsewhere; it has seemed to the writer that the student should thoroughly appreciate the principles involved in the construction of his measuring apparatus and should know the limitations of the instruments; too often he simply reads a meter and takes it for granted that the indication has a meaning which he understands; with distorted waves, various power factors, etc., there are many intricate points involved which affect the accuracy of a meter reading and it is with the intention of calling some of these points to the student's attention (they seldom are even mentioned in the various books on A.C. theory) that mathematical discussion is introduced in several places and that some experiments are given to illustrate the points covered in this discussion. The experiments serve also to bring to the student's notice the fact that our alternating current standards must generally be referred to the direct current standards and the basis on which the comparison is made.

A method for predicting the regulation of an alternator is given, in which all of the factors entering into the question are logically treated; a mathematical discussion of armature reaction in single and polyphase machines is given, for two reasons — it is not generally given in the text-books on alternating-current theory at present used, and the rigid proof is required in the discussion of armature reaction and its effect on alternator regulation.

The experiments described are designed for the use of senior students in electrical engineering, and so considerable knowledge of the laws of alternating currents is assumed. The sequence of the tests is such that the laboratory work will parallel and reinforce the lecture courses taken by the student. While the

list of experiments is doubtless incomplete, it will serve as a logical course to which other tests may be readily added at the discretion of the individual instructor, to suit his special needs or equipment. Practically all of the tests given can be readily carried out with the equipment of the average laboratory.

In the appendix are given several reproductions from ondograph or oscillograph records which serve to clear up certain involved points indicated at different parts of the text. These few illustrations indicate the kind of work for which the curve tracing apparatus is suited and represent a part of the work done by our senior students with the two instruments named. It is the writer's opinion that much more of this type of work should be incorporated in our laboratory courses.

I am glad to express my thanks to Mr. F. L. Mason, of Columbia University, who has assisted me in the preparation of the book.

J. H. M.

Columbia University,
July, 1912.

LIST OF EXPERIMENTS

ALTERNATING CURRENTS

EXPERIMENT I.

WAVE FORMS AND A.C. METERS.

ALL ordinary A.C. theory is worked out on the supposition that the voltages and currents to be considered are simple sine functions of time. It is, therefore, important to see how nearly this condition actually obtains. The fundamental connection between direct current and alternating current units is founded on the fact that the alternating current ampere flowing through a given resistance produces heat at the same rate as would the direct current ampere through the same resistance. Now the rate at which heat is produced in any conductor is given by the formula, Heat = I^2R.

In a continuous current circuit this heat is generated at a uniform rate because I is a constant. In an alternating current circuit this is not true because the current varies in magnitude with respect to time. Calling the instantaneous value of the alternating current i, the maximum value I_m, we have the equation, $i = I_m \sin \omega t$.

The heat generated by such a variable current can be expressed only by an integration, which is

Heat = $\int i^2 R dt$, where R is the resistance of the conductor in which the heat is being generated. The "effective" value of the alternating current is defined as that constant value of current, which, flowing through the same resistance, will produce in a time equal to one cycle of the alternating current, the same heat as is produced by the varying current throughout one cycle.

Calling such effective value I, we must have

$$I^2Rt = \int_0^t i^2 R dt,$$ or when R is a constant,

$$I^2t = \int_0^t i^2\, dt,$$ the time involved in the integration being that necessary for one complete cycle of current values.

This equation defines I in terms of i and evidently gives the relation, $I^2 = \text{average } i^2$, or $I = \sqrt{\text{average value of } i^2}$. In the same way is obtained $E = \sqrt{\text{mean square of } e}$, where E and e are effective and instantaneous values of voltage in the alternating current circuits.

Now an A.C. meter is always calibrated in terms of effective values and it is to investigate the validity of the above formulas that part of this test is designed.

Some scheme is necessary for obtaining wave forms of current and voltage. Such an instrument as the oscillograph or ondograph would draw the desired wave forms, but a much more accurate method is that known as the "point by point" method. This method consists in balancing the instantaneous value of alternating E.M.F. against a known continuous E.M.F. The apparatus required is a potentiometer drum, telephone receiver and a revolving switch on the shaft of the alternator supplying the waves to be investigated. The revolving switch is a disc of insulating material having imbedded in its periphery a metal strip which is connected to the alternator shaft on which the disc is mounted.

In Fig. 1, there is shown only one metal strip in the disc. This scheme will give only as many pulses per minute through the

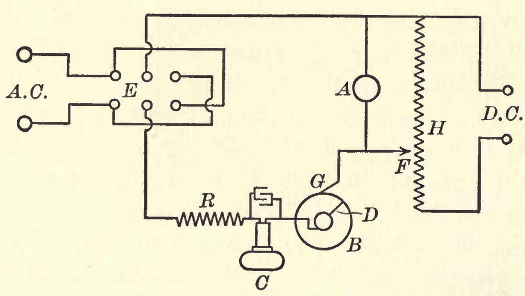


Fig. 1.

telephone as the alternator is turning revolutions per minute. As the ear does not detect easily tones of lower than perhaps sixty vibrations per second, especially in a laboratory where there are many other noises of higher tone, the disc would have to run 3600 r.p.m. for satisfactory use. But there may be more than one strip placed in the disc; in general there should be one strip for every 360° (electrical) of the armature. If the alternator

has four poles there should be two strips diametrically opposite; with an eight-pole machine there should be four strips, 90° (mechanical) apart. The disc, shown in Fig. 1, and the description of its operation, which follows, is supposed to be for a two-pole generator. A brush mounted on the alternator frame (but insulated from it) makes contact with the metal strip at the same time in each revolution of the armature. As the E.M.F. of the generator alternates synchronously with the revolution of the armature, it is evident that such a disc furnishes a means of closing a circuit at the same phase of each successive E.M.F. or current wave furnished by the alternator.

The scheme of connections is as given in Fig. 1. A resistance H serves to give a variable source of continuous E.M.F., the contact F being movable along H. The alternating E.M.F., the wave form of which is to be measured, is connected through the reversing switch E to the local circuit consisting of the telephone receiver C, revolving disc B, and part of the resistance H. When the metal strip D makes contact with the brush G, there will be acting in this local circuit two E.M.F.'s, the instantaneous value of the alternating E.M.F. and the continuous E.M.F. from the potentiometer. The sliding contact F may be moved until the voltage from the potentiometer just balances the instantaneous value of the alternating E.M.F., when no current will flow through the local circuit at the closing of the switch B, and so no noise will be heard in the telephone. (It may be necessary to reverse switch E to bring about the balance. Why?) When a balance is obtained the D.C. voltmeter A is read and so the instantaneous value of the alternating E.M.F. is obtained. By moving the brush G through 360 electrical degrees and obtaining successive balances at a sufficient number of points, the wave of alternating voltage may be plotted through an entire cycle. (A high resistance R is used to prevent overheating the telephone when the local circuit is badly unbalanced.)

Instead of connecting the local circuit to the blades of switch E, it may be connected to a two-point jack which can be connected to several receptacles, and balances obtained for several different E.M.F. waves for the same setting of the brush G. So not only forms, but relative phases of current and E.M.F.'s may be obtained. It should be noted that the effective value of any alternating E.M.F. to be measured must not be greater than 0.707 of the D.C. voltage across the potentiometer H. Why?

In any A.C. measuring instrument (with the exception of polyphase meters) the impelling force on the moving system is a varying quantity, while the resisting force (as that of a spring) is constant for a given position of the moving system.

For a given position of the moving system the following relation must hold:

time integral of varying force = time integral of constant force.

Of course, at any given instant one of the forces will be greater or less than the other so that actually the resultant force on the moving system is generally not equal to zero and the system begins to move. But as the resultant force will vary with twice the frequency of the impelling force (giving frequency of resultant force between perhaps 50 and 250 per second) it is evident that the moving system cannot oscillate with any appreciable amplitude and so will assume what seems to be a stationary position such that the above equation is true.

The curves of alternating current and E.M.F. are to be obtained by the "point by point" method, balancing the instantaneous value of the alternating E.M.F. against a variable, known, continuous E.M.F. The curve of alternating current is obtained by finding the form of the "fall of potential" curve over a known, noninductive resistance through which the alternating current is passed. Any ordinate of this curve, divided by the value of the known resistance, gives the corresponding instantaneous value of the current.

Make connections as given in Fig. 2, and take a reading for every 15° (electrical). For a noninductive resistance a lamp

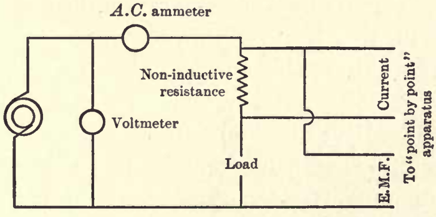


FIG. 2.

bank or noninductive rheostat may be used, the resistance of which is to be measured with direct current. Plot the instantaneous A.C. values, using time for the abscissæ. Square each value obtained and construct curve of "squared ordinates."

By planimeter, or by counting squares of section paper, or by graphical integration, obtain area of this squared curve, divide by its base and so get the "mean square" of the A.C. wave. Get the square root of this value and compare with reading of A.C. meter.

Obtain in a similar way the "average" value of the A.C. wave and calculate its "form factor." (This form factor is 1.11 for sine waves.)

With same base and on same section paper construct a sine curve having same amplitude as measured A.C. wave. This construction will show how nearly the A.C. wave used approaches a sine curve and will account for any discrepancy in the form factor.

Keep voltage and resistance constant throughout test.

EXPERIMENT II.

ALTERNATING CURRENT METERS ON CIRCUITS OF DISTORTED WAVE FORM.

THE dynamometer and hot-wire types of meter will accurately record the mean square, whatever the shape of the wave form, but other types of A.C. meters will not generally do so. The moving iron-vane type of ammeter (e.g., Weston) will only have a force proportional to the $(\text{current})^2$ so long as the iron vane maintains constant permeability. By correctly designing the meter this type will give quite accurate results on waves very much distorted.

A meter whose impelling force, due to the current flowing through its windings, varies directly with the first power of the current, if calibrated with sine wave current, will not read accurately when used with a wave form differing from sine form. The magnitude of its error will be measured by the ratio of the wave form factor to the form factor of a sine wave.

Now if the current being measured is a distorted wave it may be represented by a fundamental sine wave and a series of

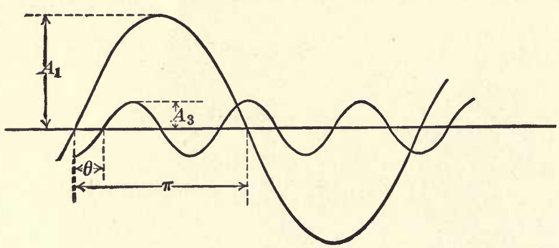


FIG. 3.

harmonics of various amplitudes. If the impelling force of the meter is proportional to the square of the current, the reading of the meter will be equal to $\sqrt{A_1^2 + A_2^2 + A_3^2 +}$, etc., where A_1, A_2, A_3, etc., represent the amplitudes (effective values), of the fundamental and the various harmonics.

Consider the case of an A.C. ammeter recording the exciting current of a transformer, the meter being one, the impelling force of which is proportional to the square of the instanta-

neous value of the current flowing through it. This wave is much distorted but may be fairly represented by the equation, $x = A_1 \cos \omega t + A_3 \cos (3\,\omega t + \theta)$, where

$A_1 =$ amplitude of fundamental,
$\omega = 2\,\pi \times$ fundamental frequency,
$A_3 =$ amplitude of third harmonic,
$\theta =$ distance on X axis between the zero values of the two waves, as in Fig. 3.

Now the meter reading, with such a current, will be proportional to the average value of the impelling force, or average value of x^2.

$$\text{Average force} = \frac{1}{\pi}\int_0^\pi (A_1 \cos \omega t + A_3 \cos (3\,\omega t + \theta))^2\, dt$$

$$= \frac{1}{\pi}\int_0^\pi A_1^2 \cos^2 \omega t\, dt + \frac{1}{\pi}\int_0^\pi A_3^2 \cos^2 (3\,\omega t + \theta)\, dt$$

$$+ \frac{1}{\pi}\int_0^\pi 2\, A_1 A_3 \cos \omega t \cos (3\,\omega t + \theta)\, dt.$$

The values of the first two integrals are quite evidently $\frac{A_1^2}{2}$ and $\frac{A_3^2}{2}$, respectively.

The third integral can be obtained by expanding the term $\cos (3\,\omega t + \theta)$.

The third integral

$$= \frac{2\, A_1 A_3}{\pi}\int_0^\pi \cos \omega t \cos (3\,\omega t + \theta)\, dt$$

$$= \frac{2\, A_1 A_3}{\pi}\int_0^\pi \cos \omega t\, (\cos 3\,\omega t \cos \theta - \sin 3\,\omega t \sin \theta)\, dt,$$

$$= K \int_0^\pi (\cos 3\,\omega t \cos \omega t)\, dt - K_1 \int_0^\pi (\sin 3\,\omega t \cos \omega t)\, dt;$$

where the K's are constants, involving A_1, A_3 and functions of θ.

We may put $\cos 3\,\omega t = \cos 2\,\omega t \cos \omega t - \sin 2\,\omega t \sin \omega t$

$$= (\cos^2 \omega t - \sin^2 \omega t) \cos \omega t - 2 \sin^2 \omega t \cos \omega t,$$

therefore,

$$\int_0^\pi (\cos 3\,\omega t \cos \omega t) dt = \left\{ \int_0^\pi \cos^4\, \omega t\, dt - 3 \int_0^\pi \sin^2 \omega t \cos^2 \omega t\, dt \right\}$$

$$= \Big[\cos^3 \omega t \sin \omega t\Big]_0^\pi = 0.$$

In the same way

$$\int_0^\pi (\sin 3\,\omega t \cos \omega t)\,dt = \int_0^\pi (3 \sin \omega t \cos^3 \omega t)\,dt - \int_0^\pi (\sin^3 \omega t \cos \omega t)\,dt$$

$$= -\left[\tfrac{3}{4}\cos^4 \omega t\right]_0^\pi - \left[\tfrac{1}{4}\sin^4 \omega t\right]_0^\pi = 0.$$

Therefore, the force acting on the moving element of the instrument $= K\left(\frac{A_1^2}{2} + \frac{A_3^2}{2}\right)$, and as the meter scale is graduated in terms of the square root of the impelling force, the reading of the meter will be $\sqrt{\frac{A_1^2}{2} + \frac{A_3^2}{2}}$.

But $\frac{A_1^2}{2} = \left(\frac{A_1}{\sqrt{2}}\right)^2 =$ (effective value)^2 of fundamental current,

and $\frac{A_3^2}{2} =$ (effective value)^2 of third harmonic current,

and the current indicated by the ammeter will be $I = \sqrt{I_1^2 + I_3^2}$, where I_1 and I_3 are the effective values of the fundamental and harmonic.

This demonstration may be easily generalized and it is found that the meter indication for any complex wave is

$$I = \sqrt{I_1^2 + I_3^2 + I_5^2 + \cdots I_n^2 \ldots}$$

As the circuit of the A.C. voltmeter is practically a noninductive resistance the current flowing through its moving element will be of exactly the same shape as the E.M.F. wave. Therefore, when acted upon by a complex wave the meter indication will be $E = \sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots\ldots}$

where E_1 = amplitude of fundamental, etc.

The validity of these two formulæ may be readily tested. Connect two alternators, of different frequencies, in series with one another. Read the voltage of each alternator and of the line, and results will be obtained as indicated in Fig. 4. Or two currents of different frequencies, the amplitude of each of which can be measured, may be used to check the current formula. Send both currents through the same ammeter and it will be found that the meter reading will be the square root of the sum of the squares of the individual currents.

In electrical quantities the different frequencies are all simply related, i.e., there are the 3rd, 5th or 7th harmonics, etc. In

case the two frequencies are very nearly alike, the integral of the cross product will have different values depending upon the interval over which the integral is taken. The force acting on the moving element of the meter will vary with a comparatively slow period and if the frequencies are close enough together the reading of the meter will be a fluctuating one, the amount of fluctuation depending upon the relative magnitude of the two frequencies. In Fig. 4, if the frequencies were 60 and 60.1, respectively, and the voltages each 100 volts, the voltage of the line would oscillate between 0 and 200.

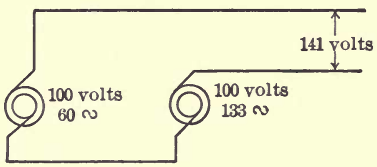


FIG. 4.

As it is easier to obtain a distorted wave of current than of E.M.F., ammeters will be used for this test. To obtain the distorted wave of current use the exciting current of a transformer, operating at about 25 per cent above normal voltage. Even though a sine wave of E.M.F. be applied to a transformer, the exciting current will be much distorted (for reasons to be discussed later, see p. 108, paragraph beginning, "The current which flows," etc.).

The value of resistance inserted in series with the transformer must not be high, otherwise the current wave will not be much

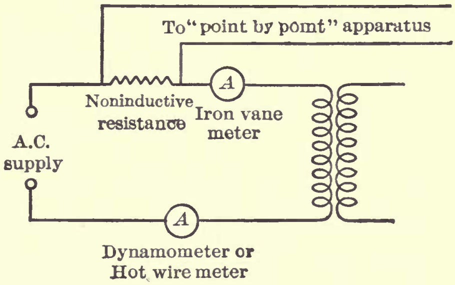


FIG. 5.

distorted. If the drop of potential over the resistance is not more than 10 per cent of the voltage impressed on the transformer, the distorted wave will be easily obtained, but if much more than this amount of resistance is inserted in the circuit the amount of current distortion obtainable will be much

reduced. The reason for this will be given in a following experiment, dealing with the wave forms of transformer quantities.

With connections as shown in Fig. 5 take a set of readings similar to those taken in Experiment 1. Use a Siemen's dynamometer or hot wire meter, for one meter and an iron-vane ammeter for the other.

Construct curve of current from instantaneous values obtained and get $\sqrt{\text{mean square}}$ by the method described in Experiment 1; compare these values with the indications of the meters; calculate the form factor of the wave.

Why should some types of A.C. meters not record accurately when measuring distorted wave forms?

Caution. — After adjusting the voltage of the alternator to about the right value, with the transformer circuit open, open the field switch of the alternator, close the transformer circuit, and then close the alternator field circuit. If this method of procedure is not used the ammeters are likely to be injured.

EXPERIMENT III.

POWER AND POWER FACTOR IN AN A.C. CIRCUIT.

In a circuit through which an alternating current is flowing the current and pressure are generally not in the same phase. The power in any circuit is the integral of the product of instantaneous values of E and I and this product, when E and I are out of phase, will be negative during a part of the cycle. During the negative part of the power cycle, current is flowing in a direction opposite to the impressed E.M.F., i.e., the circuit is feeding power back into the supply circuit.

When E and I are represented by rotating vectors it is readily seen that the power will be given by the expression $EI \cos \phi$, where ϕ is the phase difference of E and I.

Because of the occurrence of the negative loops in the power cycle, power is generally a double-frequency function. The only case when this is not true is when the current and impressed E.M.F. are exactly in phase, in which case the negative loop disappears and the power is expressed by the product EI. The indicating wattmeter, in the same way as mentioned in Experiment 1, balances an oscillating force against a steady force, and the oscillating force is sometimes negative (when $\cos \phi < 1$). The moving system of the wattmeter tries to vibrate with the frequency of the power curve, but, owing to its inertia and the rapidity of change of the impressed force, it cannot oscillate, and so assumes some intermediate position.

The current through the potential coil of the wattmeter should be in phase with the E.M.F. of the circuit being measured. This means that the potential circuit must be wound non-inductively. In case a multiplier is used with a wattmeter, care should be taken to see that it is so wound. The slight amount of inductance always present in the potential circuit introduces an error varying with the power factor of the circuit being tested; it is inappreciable on large power factors but becomes much larger as the power factor decreases. This effect is shown in Fig. 6, where the error caused by angle θ (due to inductance in potential circuit) is seen to be large when ϕ is

large and comparatively small when ϕ is small. When ϕ is small the phase relations of E and I are as shown. Representing the current by OI and the true phase of the voltage by OE, then, owing to the inductance of the potential circuit, the wattmeter acts as if the impressed E.M.F. was not OE, but OE', lagging behind OE by the angle θ, where

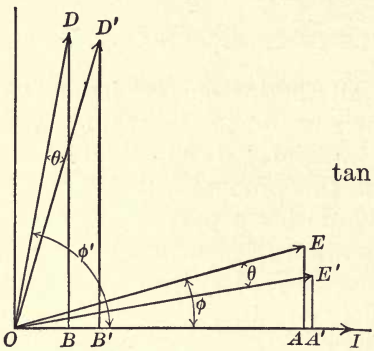


FIG. 6.

$$\tan\theta = \frac{\text{Reactance of potential coil}}{\text{Resistance of potential coil}}.$$

The wattmeter should indicate the power, $OA \times OI$, whereas it really indicates $OA' \times OI$. The percentage error is given by the fraction $\frac{AA'}{OA}$.

When the power factor of the circuit to which the wattmeter is connected is small, conditions are as indicated by OD and OD'. The percentage error is now $\frac{BB'}{OB}$, which is much larger than was the case with the load of power factor $= \cos\phi$.

Expressed analytically we have,

$$\text{True power} = EI\cos\phi,$$
$$\text{Indicated power} = EI\cos(\phi - \theta) = EI\{\cos\phi\cos\theta + \sin\phi\sin\theta\}.$$

As θ is always small, $\cos\theta$ does not appreciably differ from unity and we have

$$\text{Indicated power} = EI(\cos\phi + \sin\phi\sin\theta),$$

which shows that the error introduced varies with the sine of the characteristic angle of the circuit being measured. The indicated power is too large when the circuit being measured has a lagging current and too small when the current in the circuit leads the E.M.F.

When the question of power measurement in a circuit, having either the current or E.M.F., or both, complex quantities, is considered, the action of the wattmeter must be analyzed.

The field in such an instrument is produced by the current in the circuit, and the current in the moving coil is of the same shape as, and proportional to, the voltage of the circuit being

tested. The impelling force will then vary as the product of these two quantities.

Suppose the E.M.F. is simple harmonic

$e = E_m \cos \omega t$, and the current is complex,

$x = A_1 \cos (\omega t + \phi) + A_3 \cos (3\,\omega t + \theta)$.

What will the wattmeter read when connected to such a circuit?
Average Force

$$= \frac{1}{\pi}\int_0^{\pi} E_m \cos \omega t \,\{A_1 \cos (\omega t + \phi) + A_3 \cos (3\,\omega t + \theta)\}\, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} E_m A_1 \cos \omega t \cos (\omega t + \phi)\, dt$$

$$+ \frac{1}{\pi}\int_0^{\pi} E_m A_3 \cos \omega t \cos (3\,\omega t + \theta)\, dt.$$

Now, it has previously been shown that an integral of the form of the second term is equal to zero, therefore

$$\text{Average Force} = \frac{1}{\pi}\int_0^{\pi} E_m A_1 \cos \omega t \cos (\omega t + \phi)$$

$$= \frac{E_m}{\sqrt{2}} \frac{A_1}{\sqrt{2}} \cos \phi.$$

This analysis may be carried out for a current containing any number of harmonics, but it will be found that the wattmeter reading is equal to the product of the effective values of the E.M.F. and the fundamental current and the cosine of their phase difference. So long as one of the quantities, E.M.F., or current, is simple harmonic, the wattmeter reading is entirely independent of any upper harmonics which may exist in the other quantity.

If, however, there is a third harmonic in both E.M.F. and current, then the wattmeter reading will be proportional to

$$\frac{1}{\pi}\int_0^{\pi} \{E_1 \cos \omega t + E_3 \cos (3\,\omega t + \gamma)\}\, \{(A_1 \cos (\omega t + \phi)$$

$$+ A_3 \cos (3\,\omega t + \theta))\}\, dt.$$

Upon evaluation this integral yields two terms:

$$\text{Wattmeter reading} = \frac{E_1}{\sqrt{2}} \frac{A_1}{\sqrt{2}} \cos \phi + \frac{E_3}{\sqrt{2}} \frac{A_3}{\sqrt{2}} \cos (\gamma - \theta),$$

that is, the wattmeter reading is the sum of the watts obtained by multiplying the effective value of each voltage by the effective value of the current of the same frequency by the cosine of their phase difference.

From this simple discussion it is evident why the power factor of a circuit upon which there is impressed a simple sine curve of E.M.F., and in which there is flowing a complex current, is more or less a fictitious quantity. The ratio of watts to (volts $\times$ amperes) gives a certain number, but the significance of such number is not at once apparent. The wattmeter takes no account of the upper harmonics, but the product, volts $\times$ amperes, does involve them indirectly because the ammeter reads the (root mean square) of the amplitudes of the fundamental and all upper harmonics which may be present. The value of ϕ so obtained does not signify the phase difference of the zero points of the E.M.F. and current waves of the circuit. Neither does it signify the phase difference of the E.M.F. and the fundamental current wave. The real significance of ϕ so obtained is the phase displacement of the E.M.F. and a fictitious simple harmonic current, the effective value of which is the same as that of the complex wave, the frequency of which is the same as that of the fundamental current, and the phase of which is such that it would produce on the wattmeter the same effect as is produced by the actual complex wave.

When accuracy of measurement is desired care must be exercised in connecting the different instruments, as the errors introduced by the power consumption of the meters themselves may be appreciable. An A.C. voltmeter may use as much as 0.1 ampere or more. If the total current which the circuit is supplying is 1 ampere, the voltmeter may, if improperly connected, introduce an error of the order of 10 per cent.

Suppose a circuit as shown by full lines in Fig. 7, and that the current taken by the circuit H is 1 ampere and that the voltmeter takes 0.1 ampere. The ammeter will indicate the vector sum of the current through H and that through the voltmeter. Even though the magnitude of the voltmeter current is known, the relative phase of the circuit current and voltmeter current is not generally known, so that the ammeter reading cannot be corrected.

Now, if the voltmeter is so connected that its current does not flow through the ammeter (as shown by the dotted line in Fig. 7), the reading of A will be the true current through H. But now

the voltmeter reading is not the voltage across circuit H, but something greater because there must be some drop of potential through the ammeter. This drop will generally be small and in most circuits would probably be inappreciable. If the current through H is very large compared to the voltmeter current the connections may be made as shown in the full lines of Fig. 7 and the error of measurement will be small.

When a wattmeter is used for measuring the power in a circuit the same precautions must be observed in connecting its potential coil as were noted above for the voltmeter connection. If the wattmeter is connected, as shown in the full lines of Fig. 8, the

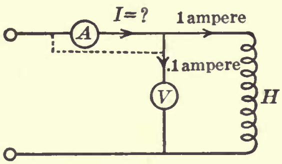


Fig. 7.

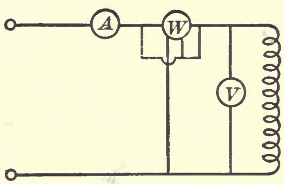


Fig. 8.

reading of the meter represents the power used in coil H plus the power used in the potential coil itself. If this loss is known it may be subtracted from the wattmeter reading; if

E = potential difference applied to potential coil
R = resistance of potential coil, then the

correction (amount to be subtracted from wattmeter reading) $= \frac{E^2}{R}$.

In case the wattmeter is of the compensated type no error of this sort is incurred and so no correction need be applied. In this type of meter the coil generating the magnetic field, in which the moving coil lies, and with which the moving coil reacts to produce deflection of the meter, is wound of two wires side by side, one of them of large enough cross section to carry the rated current capacity of the meter and the other of fine wire through which the potential-circuit current flows. The internal connections of the coils are as shown in Fig. 9. If the meter is connected to the circuit, as shown in the full lines of Fig. 8, it is evident that the M.M.F. due to coil A is produced by the current $I + i$, where I = current through circuit H, i = current through potential circuit of meter. Now the compensating winding is so con-

nected that when the meter is properly connected to circuit H, i.e., in such fashion that the meter gives a positive deflection, the current i through the compensating winding produces a M.M.F. in direction opposite to that produced by coil A. The resultant M.M.F. and hence the field influencing the moving coil C, is due to current $\{(I + i) - i\}$ or I. The reading of the wattmeter is, therefore, independent of the current taken by the potential coil and the meter should be connected as shown in the full lines of Fig. 8. If the potential circuit is connected as shown by the dotted line of Fig. 8 the wattmeter will indicate inaccurately, as the field of the meter will be proportional to $(I - i)$ instead of I.

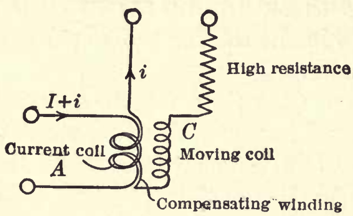


FIG. 9.

With connections given in Fig. 10, keeping voltage and frequency of supply constant, obtain the curves of current and voltage by the "point by point" method. Read also the ammeter, voltmeter and wattmeter. Adjust the circuit so that

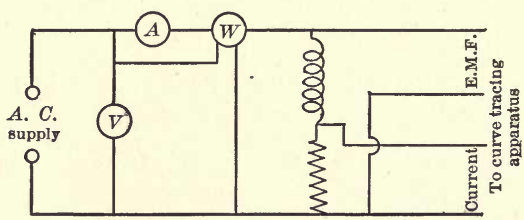


FIG. 10.

$\cos \phi$ is 0.8 or less. Take one set of curves using an inductance coil having an air core, in which case the current will be a sine curve and the measured phase difference, as obtained from curve sheet, should check with value calculated from the ratio, $\cos \phi = \dfrac{\text{watts}}{EI}\cdot$ Then take a set of curves using an iron-core inductance, forcing through it enough current to thoroughly saturate the iron core, under which condition the current curve will be much distorted. The measured ϕ from the curves will not now check with the value obtained from the meters. This fact emphasizes the necessity of confining the simpler calculations of A.C. quantities to sinusoidal functions.

With the data obtained, construct curves of current and voltage and from these two construct the curve of power. Get the area of the power curve (by planimeter or otherwise) and compare with the wattmeter reading. Scale off the angular displacement between zero points of current and E.M.F. waves; find the cosine of this angle and compare with the power factor obtained by the ordinary formula.

EXPERIMENT IV.

MEASUREMENT OF SELF-INDUCTION AND EFFECTIVE RESISTANCE.

WHENEVER the current through a conductor is varied a counter E.M.F. is set up in the conductor due to the change in strength of the magnetic field surrounding it. The magnitude of this C.E.M.F. depends upon the constants of the circuit (number of turns and the reluctance of the magnetic circuit) and upon the rate of change of the current.

The C.E.M.F. may be expressed by the equation,

$$\text{C.E.M.F.} = (\text{constant}) \times \left(\frac{di}{dt}\right);$$

this constant for any given circuit is called the coefficient of self-induction, generally designated by L. It is to be noted that L is not a constant when the reluctance of the magnetic circuit is variable, as e.g., when there is iron in the magnetic circuit, in which case L will depend upon the value of the current. From this it will be noted that L will vary throughout the A.C. cycle, because the permeability of the iron varies with the instantaneous value of the alternating current. L is, however, ordinarily obtained in terms of effective E and I, in which case it is treated as a constant throughout the cycle.

If there were no resistance in the circuit considered we might write:

$$\text{Impressed voltage} = L\frac{di}{dt}.$$

As all circuits have resistance this equation must include the resistance drop, so we have

$$\text{Impressed voltage} = L\frac{di}{dt} + Ri.$$

The fact that i is a sine function of time and that $\frac{di}{dt}$ is a cosine function shows that the two components of the C.E.M.F. must be added as vectors at right angles to give the impressed E.M.F. as a vector.

Considering the vector diagram and using the ordinary notation we have $E = I\sqrt{(2\pi fL)^2 + R^2}$, where E and I are effective values.

The power consumption in the circuit is independent of the inductance component of the C.E.M.F. as this component is at right angles to the phase of the current, hence the resistance component, IR, must be such a quantity that, when multiplied by the current, it gives the total power consumption in the circuit. When there is iron in the magnetic circuit, power will be used up due to hysteresis and eddy currents in the iron. Hence the power consumption will be greater than I^2r (r being the ohmic resistance of the circuit) and when we write

$$\text{Power consumed} = I^2R,$$

R stands for something more than ohmic resistance, and is called the "effective" resistance of the circuit; to get this effective resistance of any circuit measure the power in watts by a wattmeter and divide by the square of the current. This effective resistance will generally be greater than the ohmic resistance; it may, however, be less than the ohmic resistance when two mutually inductive circuits are considered, moving with respect to one another, as in the case of the induction generator.

Measure the effective resistance and coefficient of self-induction of two inductance coils (one having air core and one with magnetic circuit of iron) at several values of current and frequency. Account for changes in their values? Measure the ohmic resistance of the coil and compare with the effective resistance. Calculate the power factor of the circuit.

With a given impressed voltage on the coil would you expect L and R to vary with the frequency. Why?

EXPERIMENT V.

MEASUREMENT OF COEFFICIENT OF MUTUAL INDUCTION.

Whenever the magnetic flux through a circuit varies there will be an E.M.F. set up in the circuit. The flux may be generated by current in the coil considered or by another coil so situated with respect to the one considered that part of its flux threads the second circuit. It is then clear that whenever the current in the first coil is varied, thereby changing the strength of its magnetic field, there will be an E.M.F. set up in the second coil. The magnitude of this E.M.F. will depend upon the number of turns in the two coils, the position of one with respect to the other, reluctance of path of the mutual flux and the rate of change of current in the first. The value of the E.M.F. generated in the second coil may be written, $e = M\frac{di}{dt}$, where i is the current in the first coil and M is called the coefficient of mutual induction of the two coils.

When the reluctance of the path of the mutual flux is a constant quantity, then the coefficient M will be a constant for all values of current, and will be the same, whichever coil is used for creating the magnetic field. If the magnetic circuit is composed in part or altogether of iron, and the two coils have different numbers of turns, then the value of M determined will be in general different when each coil in turn is used to create the field; but, if, when both measurements are taken, the field is at the same density then M will be the same in both cases. It is self-evident that if we desire M to be as large as possible the two coils should be so situated that all the field produced by the first coil threads the second. This is never possible but is approximated in the constant potential transformer, where the two coils are generally either concentric or laminated, the sections of the two coils being interspersed.

If a transformer is desired having a variable M (as in the case of the constant current transformer) then a leakage path of variable reluctance is used, which prevents more or less of the field of the first coil from linking with the second. In measuring

M, effective values of E.M.F. and current are read, in terms of which the equation for induced E.M.F. becomes

$$\text{E.M.F.}_{\cdot 1} = M\,2\pi f I_2.$$

As the coefficients of self-induction of the two coils and their coefficient of mutual induction all depend upon the field strength and number of turns of the coils it is evident that some mathematical relation must be obtainable, which will express one in terms of the others.

If n_1 = number of turns in coil No. 1,

n_2 = number of turns in coil No. 2,

ϕ_1 = flux per ampere, through coil No. 1, produced by current in No. 1,

ϕ_2 = flux per ampere, through coil No. 2, produced by current in No. 2,

K = coefficient of leakage between the two coils, i.e., the fraction by which ϕ_1, e.g., must be multiplied to give the flux through coil No. 2 due to current in coil No. 1, or vice versa,

$$L_1 = n_1\phi_1,$$

$$L_2 = n_2\phi_2,$$

$$M = n_1K\phi_2 = n_2K\phi_1,$$

$$L_1L_2 = n_1n_2\phi_1\phi_2,$$

$$M^2 = n_1n_2\phi_1\phi_2K^2 \text{ or } M = K\sqrt{L_1L_2}.$$

So the coefficient of leakage between the two coils may be determined by measuring M, L_1 and L_2.

Using first two separate coils with air cores, find out how M varies with current, frequency and relative position of coils.

Then use two coils on the same iron circuit and make similar tests. If possible, introduce a better leakage path between the two coils and again measure M.

EXPERIMENT VI.

TO MEASURE THE CAPACITY OF A CONDENSER.

THE most natural method to employ in this test would be to measure the two quantities by which capacity is defined. If we measure the charge Q, required to bring the condenser plates to a difference of potential V, then by definition, $C = \frac{Q}{V}$. This method is used where C is small and a ballistic galvanometer is available for measuring Q. In case C is as large as a few microfarads or more, a convenient way of measuring it is to determine the charging current when a known alternating E.M.F. is applied to its terminals.

If we apply a varying E.M.F. to the condenser the fundamental equation becomes

$$C\frac{dV}{dt} = \frac{dQ}{dt} = i \text{ or } i = C\frac{dV}{dt}.$$

When $\frac{dV}{dt} = 2\,\pi f E_m \sin 2\,\pi f t$, then as i must be a function similar to $\frac{dV}{dt}$, we have

$$i = I_m \sin 2\,\pi f t,$$

and

$$C\,2\,\pi f E_m \sin 2\,\pi f t = I_m \sin 2\,\pi f t \text{ or}$$

in effective values $2\,\pi f C E = I$; hence, if we measure E and I, C can be calculated when f is known.

If an accurate determination of C is desired an electrostatic voltmeter of low capacity must be employed to measure E. If a closed-circuit voltmeter is used, one of two errors may be introduced. Reading the charging current when the voltmeter is shunted across the condenser may give too large a current due to that taken by the voltmeter in parallel with the condenser, or may give too small a value if the voltmeter has an appreciable inductance.* An inductance and a condenser in parallel with each other may be so proportioned that practically all of the

* This condition, of course, never occurs with an ordinary commercial meter.

charging current for the condenser is furnished by the inductance and will not be read by an ammeter in the supply circuit.

A perfect condenser would use no power, but in practice the dielectrics are imperfect and the continually reversing polarization causes an energy loss similar to the hysteresis loss in iron subjected to magnetic reversals. There is also more or less mechanical vibration of the condenser plates which necessitates power consumption; also there occurs actual leakage of current from one plate of the condenser to the other and this leakage causes energy loss in the condenser dielectric. This dielectric loss is generally very small, so that the charging current will lead the impressed voltage by an angle of between 85° and 90°. The dielectric loss will heat a condenser and care should be exercised that a condenser intended for temporary use (starting condenser for single-phase induction motor, e.g.) is not left connected to the circuit longer than necessary. Also a condenser should not be subjected to a higher voltage than that for which it was designed, as the increased losses may cause the dielectric to deteriorate if not actually to puncture. In case the dielectric used in the condenser is paraffine, the danger from overheating the condenser is very evident. The electrical resistance of paraffine decreases enormously as its temperature rises; its resistance at 90°C., e.g., is only one-fortieth of the value at 40°C.; if the first heating is due to actual leakage current through the dielectric, the condenser is almost sure to break down if left on the circuit for a sufficient length of time, as the heating effect is cumulative; an increased temperature produces an increase in the rate at which power is being used in the dielectric and so the temperature increases until the dielectric is punctured.

Owing to the fact that a dielectric such as paraffine has a lag angle (i.e., its polarization lags somewhat behind the polarizing force) a slight decrease in capacity may be expected as the frequency increases, because the charge does not have sufficient time to "soak in" or effect the same degree of polarization as it would if the E.M.F. was applied continuously in the same direction. Certain paraffine condensers showed a decrease from 38.5 m.f. to 35.8 m.f. as the frequency was increased from 40 cycles to 80 cycles.

When the dielectric used has a small lag angle then very little power is used in charging and discharging the condenser. Such is the case with mica. Paraffine paper condensers give, however,

considerable loss, certain condensers tested giving results as shown in Fig. 11. It will be observed that the loss follows somewhat the same variations as would the hysteresis loss in a piece of iron subjected to various m.m.f.'s and frequencies.

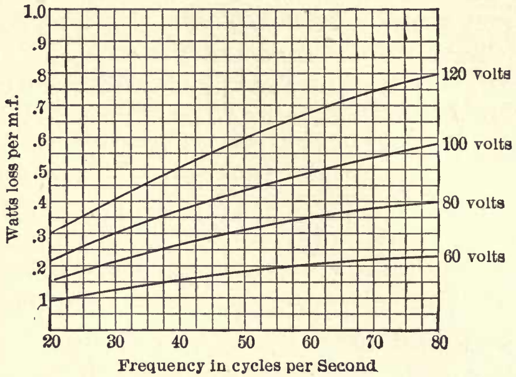


FIG. 11.

By means of an A.C. ammeter and voltmeter determine the capacity of a condenser and make proper tests to see if the capacity changes with frequency and voltage. By use of a wattmeter determine the power used in the condenser at various voltages and frequencies and calculate the phase angle of the charging current. The wattmeter reading must be corrected for the error incurred by the inductance of its potential circuit, if accurate results are to be obtained. This error is explained in Experiment 3.

EXPERIMENT VII.

STUDY OF THE REACTIONS IN AN ALTERNATING-CURRENT CIRCUIT.*

In a direct-current circuit containing nothing but a resistance the only reaction opposing the impressed E.M.F. is the IR drop, and so we have the reaction equation in the familiar form of Ohm's law, i.e., $E = IR$.

If we have a circuit containing, besides resistance, a counter E.M.F., such as the circuit of a storage battery being charged, then the reaction equation must be written in the form

$$E = IR + E_{\text{counter}},$$

or in its general form, known as Kirchhoff's second law, it is

$$E = \Sigma\, IR + \Sigma \text{ counter E.M.F.'s.}$$

It should be noticed that this method of analyzing the problem of the electric circuit, i.e., equating the impressed force to the sum of the reacting forces, is much more logical than the one usually employed in which the current flowing is put equal to the impressed force divided by the resistance; it should be borne in mind that the quantities which can be measured experimentally are reactions. The resistance of a wire cannot be measured directly, but the quantity measured is the drop in pressure, or resistance reaction, caused by current flowing through the conductor.

In a Wheatstone bridge the actual resistance of the unknown conductor is not measured. A current is sent through the conductor and the bridge is so adjusted that the resistance reaction of the unknown resistance is equal, or proportional, to the reaction due to the same current passing through a conductor of known resistance. In measuring resistance by the "fall of

* The idea of attacking all A.C. problems by means of the fundamental principle that, in any electrical circuit, the impressed force must be equal to the sum of all the reactions in the circuit, I owe to Prof. M. I. Pupin. I have become convinced of the utility of the reaction conception in solving electrical problems by actually using it in the solution of some original problems on which I have been recently working with Prof. Pupin.

potential" method, using voltmeter and ammeter, the actual resistance is not measured. The resistance reaction is the quantity measured.

In the alternating-current circuit containing resistance, inductance and capacity in series there are three reactions to consider.

The resistance drop, iR, is at every instant proportional to the current, and, as it opposes the flow of current, it is 180° out of phase with the current. The quantity R will not be the actual ohmic resistance of the circuit but will be the "effective" resistance as described in Experiment 4.

The inductance reaction of the circuit is proportional to the time rate of change of the current and is given by the expression $-L\frac{di}{dt}$. From consideration of a sine wave it is evident that when the current is zero and increasing the inductance reaction will be a negative maximum, i.e., it **lags behind** the current by 90°. Hence the component of the impressed E.M.F. which must be used in overcoming this reacting force will **lead** the current by 90°.

The magnitude of the counter E.M.F. of the capacity is given by the formula (Experiment 6)

$$\frac{dV}{dt} = \frac{i}{C} \quad \text{or} \quad V = \frac{1}{C}\int i\,dt.$$

When the action of the condenser is analyzed it is seen that this reacting E.M.F. has its maximum negative value at the end of a positive alternation of the current. This is evident because the condenser pressure acts to decrease its charge, and at the end of a positive alternation of current the charge will be a positive maximum. So that the reaction of the condenser will **lead** the current by 90° and the component of the impressed E.M.F. to overcome this reaction will **lag** behind the current by 90°.

The general reaction equation then of the alternating-current circuit is

$$E = L\frac{di}{dt} + Ri + \frac{Q}{C},$$

and as $i = \frac{dQ}{dt}$ the differential equation of reaction becomes

$$E = L\,\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C}$$

from which the value of current may be obtained for any

circuit of known constants, when the impressed E.M.F. and frequency are given.

From this analysis it is seen that the impressed E.M.F., treated as a vector, will have three components, IR in phase with current, $2\pi fLI$ leading current by 90°, and $\frac{I}{2\pi fC}$ lagging 90° behind the current. As a matter of fact, the total reaction of neither the inductance nor condenser will be exactly 90° out of phase with the current because of losses which occur in them, which losses must be supplied by a component of the impressed force in phase with the current. But whatever the relative phases of the reacting forces, they must always add up as vectors to give the negative of the impressed E.M.F. plotted as a vector.

Connect to an A.C. supply a noninductive resistance, an inductance and a condenser, in series with an ammeter and the current coil of wattmeter. With an A.C. voltmeter measure the impressed E.M.F. and drop in potential across each part of the circuit. Also read the power consumption in the whole circuit and in each part and read current. Keep impressed voltage and frequency constant while volts and watts are being read; take another set of readings, leaving L, R and C the same but with f changed and see how the different reactions change with a change in f. Take another set of readings after having adjusted L, R and C to different values.

With the readings obtained construct vector diagrams to see whether or not the component E.M.F.'s add to give the impressed E.M.F. in both magnitude and phase.

EXPERIMENT VIII.

OHM'S LAW APPLIED TO THE ALTERNATING-CURRENT CIRCUIT.

THE quantity by which the impressed A.C. voltage of any circuit must be divided to give as quotient the current flowing in the circuit is called the impedance of the circuit and generally designated by Z. This quantity is generally expressed in ohms, although it really involves the units of inductance and capacity as well as the ohm. By definition of Z we have, $IZ = E$, but in the preceding experiment it was shown that E was generally made up of three components to balance the three reactions in the general A.C. circuit. Using the prefixed letter j to designate vectors rotated 90° in a clockwise direction and $-j$ to indicate counter clockwise rotation, the equation of reactions may be written

$$E = IR + (-j\,(2\,\pi fLI)) + j\left(\frac{I}{2\,\pi fC}\right),$$

$$= I\left(R - j\left(2\,\pi fL - \frac{I}{2\,\pi fC}\right)\right) = IZ.$$

From this it may be seen that Z, considered from the standpoint of reactions, is a complex quantity and has direction, but as Z is ordinarily used as a simple magnitude, and not as a vector, it is necessary to find the magnitude of the above vector and call this value Z.

As the two components of Z in the above expression are at right angles to one another it is evident that the magnitude of Z is given by the expression

$$Z = \sqrt{R^2 + \left(2\,\pi fL - \frac{1}{2\,\pi fC}\right)^2},$$

and so for the A.C. circuit containing resistance, inductance and capacity in series, Ohm's law becomes

$$I = \frac{E}{\sqrt{R^2 + \left(2\,\pi fL - \frac{1}{2\,\pi fC}\right)^2}} = \frac{E}{\sqrt{R^2 + X^2}},$$

and the phase difference of I and E is given by equation

$$\tan \phi = \frac{\left(2\,\pi f L - \dfrac{1}{2\,\pi f C}\right)}{R} = \frac{X}{R}.$$

When dealing with circuits having two or more branches in parallel it becomes convenient to introduce some new terms. Corresponding to the factors, resistance, reactance and impedance, we shall have the terms conductivity, susceptance and admittance, designated respectively by the symbols g, b and Y. The significance and use of these terms will now be taken up.

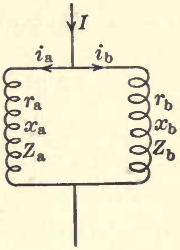


FIG. 12.

Referring to Fig. 12, which shows two inductances in parallel, it is evident that the line current I is equal to the vector sum of the branch currents i_a and i_b. If the potential difference of the extremities of the parallel path is E, it is seen at once that

$$i_a = \frac{E}{Z_a} = \frac{E}{\sqrt{r_a^2 + x_a^2}}$$

and that it is made up of two components, one in phase and one 90° out of phase with E.

$$i_a \text{ (in phase)} = i_a \cos \phi = i_a \frac{r_a}{Z_a} = \frac{E r_a}{Z_a^2}.$$

$$i_a \text{ (90° out of phase)} = i_a \sin \phi = \frac{E x_a}{Z_a^2}.$$

Collecting these expressions we have

$$i_a \text{ (total)} = \frac{E}{Z_a} = E Y_a,$$

where Y_a is called admittance of circuit a;

$$i_a \text{ (in phase)} = \frac{E r_a}{Z_a^2} = E g_a,$$

where g_a is called conductance of circuit a;

$$i_a \text{ (90° out of phase)} = \frac{E x_a}{Z_a^2} = E b_a,$$

where b_a is called susceptance of circuit a.

If now E is taken as having a magnitude of one volt the meanings of Y, g and b, become apparent. They are the total

current, in-phase current, and out-of-phase current, respectively, per unit E.M.F. impressed on the circuit.

In the same way as given above Y_b, g_b and b_b are obtained. As the line current is the vector sum of currents through the separate paths, and it is easiest to treat vector quantities by using their X and Y components, we shall have

$$\text{Line current (in phase)} = E(g_a + g_b) = EG.$$

$$\text{Line current (out of phase)} = E(b_a + b_b) = EB.$$

$$\text{Lag angle of line current} = \tan^{-1}\frac{B}{G}\cdot$$

The same method of adding conductances and susceptances is employed for any number of paths in parallel.

If these parallel circuits are in turn in series with other circuits it becomes necessary to express G and B in terms of resistance and reactance.

For any circuit the in-phase current is given by the expression

$$I \text{ (in phase)} = \frac{E}{Z}\cos\phi = \frac{ER}{Z^2},$$

$$I \text{ (out of phase)} = \frac{E}{Z}\sin\phi = \frac{EX}{Z^2},$$

$$I \text{ (total)} = \frac{E}{Z} = EY.$$

From this is obtained $G = \frac{R}{Z^2}$ and $B = \frac{X}{Z^2}$ where R and X represent the resistance and reactance of the combined parallel paths. Hence, to reduce the two parallel paths to an equivalent single path having characteristics R and X, we have:

$$R = GZ^2 = \frac{(g_a + g_b)}{Y^2} = \frac{\frac{r_a}{Z_a^2} + \frac{r_b}{Z_b^2}}{Y^2}$$

$$= \frac{\frac{r_a}{Z_a^2} + \frac{r_b}{Z_b^2}}{\{(g_a + g_b)^2 + (b_a + b_b)^2\}} = \frac{\frac{r_a}{Z_a^2} + \frac{r_b}{Z_b^2}}{\left(\frac{r_a}{Z_a^2} + \frac{r_b}{Z_b^2}\right)^2 + \left(\frac{x_a}{Z_a^2} + \frac{x_b}{Z_b^2}\right)^2}.$$

In a similar fashion

$$X = \frac{\frac{x_a}{Z_a^2} + \frac{x_b}{Z_b^2}}{\left(\frac{r_a}{Z_a^2} + \frac{r_b}{Z_b^2}\right)^2 + \left(\frac{x_a}{Z_a^2} + \frac{x_b}{Z_b^2}\right)^2}.$$

If a third circuit c is in series with this parallel path as shown in Fig. 13, the current $I = \frac{E}{Z} = \frac{E}{\sqrt{(r_c + R)^2 + (x_c + X)^2}}$. In case a condenser is used in any part of the circuit its reactance, x, must, of course, be taken as negative.

The solution of parallel circuits can also be obtained by vector construction and generally this is easier than the analytical method given above. The solution of parallel circuits by the use of graphical methods is, in fact, so much easier than by the analytical method that the latter is only to be recommended in the simplest cases.

The graphical method of solution is based on the fact that the two-branch currents may be added as vectors to give the line current in magnitude and phase; no matter how many circuits join at some branch point, the vector sum of all currents flowing away from the point must equal that of the currents flowing to it. If the currents of branches a and b (Fig. 12), are added vectorially, as in Fig. 14, the line current is given by the vector

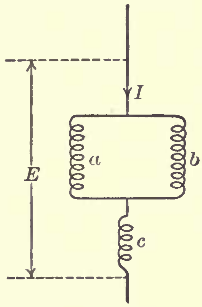


FIG. 13.

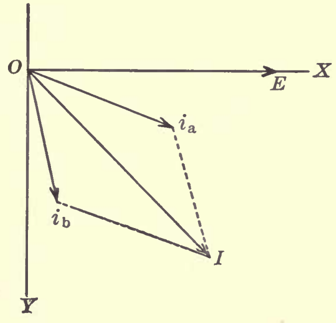


FIG. 14.

OI, both in magnitude and proper phase with respect to the voltage vector OE. If now the currents i_a and i_b are taken of proper magnitude to represent the respective current per unit potential difference on the two circuits, then OI is the line current per volt impressed on the circuit. Now the reciprocal of the current per volt OI will give the impedance of the combined paths in ohms.

By adding vectorially the " amperes per volt " of each circuit connected to the branch-point considered, the " line current per volt " is obtained and the reciprocal of this quantity is the

impedance in ohms; the components of this impedance in the Y and X axis give the reactance and resistance of the parallel circuit; to these components may be added the reactance and resistance of whatever circuit is in series with the parallel path, as c (Fig. 13), and the constants for the complete circuit are thus obtained.

With known values of L, R and C, arrange series, series-parallel and parallel circuits as shown in Fig. 15. With suitable voltmeter, ammeter and wattmeter measure the impressed E.M.F.,

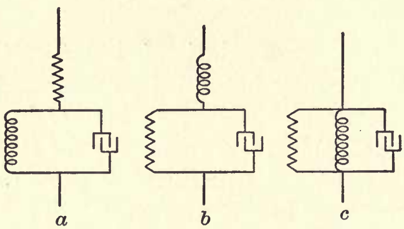


FIG. 15.

the line current and power used in the combined circuit. From the known constants of the different parts of the circuit calculate what current should flow under the pressure E and its phase with respect to E, for each of the three cases, using one of the methods just described. Compare with the measured values.

In this test it is well to use an air core inductance, otherwise L must be measured for each value of the current used in the experiment.

EXPERIMENT IX.

CIRCLE DIAGRAM FOR CIRCUIT CONTAINING RESISTANCE AND REACTANCE.

Just as it is possible to predict the characteristics of direct-current machines when certain constants of the machine are known, so can the behavior of alternating-current machinery be predetermined.

For such machines as the induction motor, series motor and transformer, it is not necessary to use the method of calculation to predetermine the characteristics of the machine, but they may be determined graphically from the "circle diagram." As this diagram is of considerable importance in experimental work it is necessary that the validity of the construction be tested.

The fundamental idea involved in the circle diagram is this: If any alternating E.M.F. of constant magnitude and frequency be applied to a circuit containing resistance and reactance in series, either one of which is varied (the other remaining constant), then the locus of the current flowing in the circuit will be a circle. That this is true may be shown mathematically as follows:

$$I = \frac{E}{\sqrt{R^2 + X^2}},$$

where X designates whatever reactance (inductance or capacity) the circuit may contain and this reactance is to be a constant value while R will vary. We may write

$$I = \frac{E}{X} \times \frac{X}{\sqrt{R^2 + X^2}} = \frac{E}{X} \cos \theta,$$

where $\frac{E}{X}$ is a constant quantity and θ is the complement of the angle between current and E.M.F. Such an equation is, of course, that of a circle $\left(\text{of diameter} = \frac{E}{X}\right)$, expressed in polar coördinates.

If then R is varied through a wide range of values, I will be given by the series of chords, drawn from one end of the diameter $\left(\text{of magnitude } \frac{E}{X}\right)$, to the circle constructed on this diameter as in Fig. 16.

As θ in the above equation is the complement of the angle between current and impressed E.M.F. the proper phase relation between I and E will be given by letting E be represented by a perpendicular constructed on that end of the diameter from which the chords are drawn.

The diameter of the circle will be the maximum possible value of the current, i.e., when R is reduced to zero, in which case the phase difference of E and I becomes 90°. This is as it should

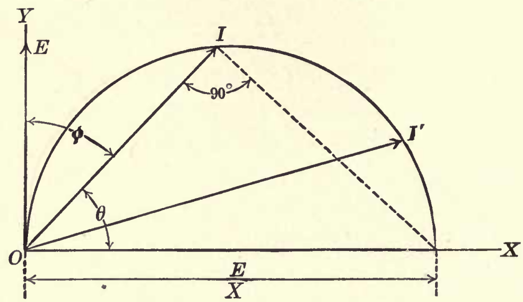


FIG. 16.

be, because no power can be used in a circuit containing only reactance. The projections of the vector I upon the diameter and upon the vector E will give its respective wattless and power components.

Connect a known air-core inductance in series with a variable noninductive resistance which can be changed throughout a wide range.* A suitable ammeter and wattmeter must be used to get the current and power used in the circuit and a voltmeter to read the impressed voltage and see that it remains constant.

Vary R through as wide a range as possible (reducing it to as near zero as is safe with the apparatus being used), reading amperes and watts for each adjustment of R.

Make the same run with an iron core inductance, in which case L will generally decrease with an increase of current.

Calculate the value of X and construct a semicircle on a diameter of the magnitude $\frac{E}{X}$. Lay off E (to any convenient scale) perpendicular to this diameter at its left extremity. From the same end of the diameter lay off a series of lines

* As an air-core coil does not have much inductance it will probably be best to use an alternating current of the highest frequency obtainable.

making with E angles whose cosines are calculated from the observations of watts and volt-amperes. Upon each line lay off (to the same scale as the diameter of the semicircle) its respective value of current to see how nearly the semicircle comes to being the locus of the current under the different conditions, both for the constant L and variable L.

What would be the effect upon the diagram of a decreasing reactance with increasing current? Such an effect actually exists in A. C. machinery; the demagnetizing effect of the rotor currents of an induction motor act to produce an apparent decrease in the inductance as the load is increased.

EXPERIMENT X.

FREE AND FORCED VIBRATIONS; RESONANCE IN A CIRCUIT CONTAINING RESISTANCE, INDUCTANCE AND CAPACITY.

Any system, mechanical or electrical, having concentrated mass and an elastic restoring force proportional to the displacement, will be capable of oscillating continually in one definite frequency, executing simple harmonic motion. Translated into electrical terminology, this means that a condenser and inductance, connected in series, as shown in Fig. 17, the condenser being charged, form a circuit in which, when the switch A is closed, current will surge back and forth with a definite frequency and will be a sine function of the time. At that time when the condenser is fully charged there is no current in the circuit and all of the energy exists as potential energy of the charged condenser. A quarter of a cycle later the condenser is discharged and so possesses no energy, but at this time the current, and hence the magnetic field of the inductance, will have a maximum value, and all of the previous potential energy of the condenser will be stored as kinetic energy of the magnetic field. If there is no resistance (effective) in the circuit (i.e., no energy is dissipated either as heat or Hertzian waves) the circuit will continually oscillate, the frequency of oscillation being fixed by the constants of the circuit as will now be shown:

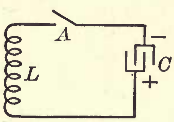


Fig. 17.

If Q = charge on condenser before switch is closed,
i = current in circuit at any time,
L = coefficient of self-induction of the coil,
C = capacity of condenser,

we have, as soon as the switch is closed and condenser begins to discharge, $i = \frac{dQ}{dt}$.

As the sum of the two reactions in the circuit must equal the impressed force, and this is zero,

$$L\frac{di}{dt} + \frac{Q}{C} = 0, \text{ or } L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0.$$

The solution of this differential is obtained by assuming $Q = A \cos \omega t$ and so $\frac{d^2Q}{dt^2} = -A\omega^2 \cos \omega t$.

Substituting these values in the original equation

$$-LA\omega^2 \cos \omega t + \frac{A}{C} \cos \omega t = 0, \text{ from which}$$

$$-LA\omega^2 + \frac{A}{C} = 0;\ \omega^2 = \frac{1}{LC},\ \text{or}\ \omega = \frac{1}{\sqrt{LC}},$$

so that, as $2\pi f = \omega$, we have $f = \frac{1}{2\pi\sqrt{LC}}$ as the frequency of the free vibrations in such a circuit. Another solution may be obtained by putting $Q = B \sin \omega t$, and, therefore, the complete solution becomes

$$i = \frac{dQ}{dt} = -\omega A \sin \omega t + \omega B \cos \omega t.$$

To get the value of the two constants, A and B, we will assume that when

$$t = 0 \text{ (i.e., before switch is closed)}, i = 0,$$

so that

$$0 = 0 + \omega B \cos \omega t \quad \text{or} \quad B = 0.$$

If at time $\frac{\pi}{2}$ the value of i is called I_m,

$$I_m = -\omega A \text{ or } A = -\frac{I_m}{\omega},$$

so that the particular solution becomes

$$i = I_m \sin\left(\sqrt{\frac{1}{LC}}\right) t.$$

Of course it is impossible to so construct a circuit that no energy is dissipated while the system is oscillating. It is, therefore, necessary to introduce into the equation of reactions a dissipative term, proportional to the current. The circuit, then depicted by Fig. 18, will have for its equation of reactions

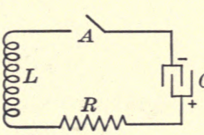


FIG. 18.

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0.$$

The solution of such an equation requires more mathematics than is thought well to introduce in a text intended for laboratory

use, but the solution is given:

$$i = Ae^{-\frac{Rt}{2L}}\cos\left\{\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\right)t + \beta\right\},$$

where β depends upon the time from which t is reckoned.

This solution is somewhat different from the previous one. The exponential term will evidently produce damping, so that the vibrations gradually die out, their energy being dissipated from the system in the form of heat or electric waves. The frequency of the vibration is given by the cosine term so that

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}},$$

which is a slower vibration than that obtained for the non-damped circuit. In fact, if the resistance is too high, no oscillation of current takes place; the current dies out without ever reversing its direction.

It is to be noticed that the damping and difference in frequency from the first case are not affected by the resistance alone but by the ratio of the resistance to the inductance.*

The general case is now considered. A circuit as shown in Fig. 11 has impressed upon it an alternating E.M.F. The equation of reactions becomes

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E\cos\omega t.$$

The complete solution of this equation gives two terms, one having a frequency equal to that of the impressed force and the other having a frequency equal to that just obtained for the free vibration of such a system.

The complete solution is

$$i = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\sin(\omega t - \epsilon) + Ae^{-\frac{Rt}{2L}}\cos\left\{\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}t + \beta\right\}.$$

The angle ϵ depends upon the constants of the circuit and A depends upon the instantaneous value of the impressed force when the switch is closed on the circuit. Its value, however, will never be greater than

$$\frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

* For curves showing this oscillatory discharge and the effect of resistance, see Appendix, Plates 1 and 2.

As will be seen, the second term soon becomes negligible because of its damping coefficient, so that for the steady state (which is the condition investigated in the laboratory) the solution reduces to the well-known form:

$$i = \frac{E}{Z} \sin (\omega t - \epsilon).$$

It is interesting, however, to investigate the significance of the second member of the solution. In the steady state, the current i has a certain fixed amplitude and a certain angle of displacement, with respect to the impressed force. In Fig. 19, the full line marked E represents the impressed force, the full line marked i represents the steady value of the current in its

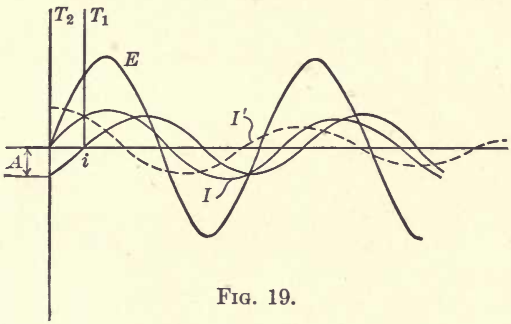


FIG. 19.

proper phase with respect to E. It is quite evident that at the time of closing the switch which connects the source of E.M.F. to the circuit, the current must have zero value. If the switch is closed at time T_1, the current i will have its proper zero value and proper phase with respect to E, and if the condenser should happen to be charged to the potential difference that normally exists across it at that part of the cycle indicated by T_1, then the transient or exponential term will reduce to zero; it never exists. In general, the switch will not be closed at this time but at some such time as marked T_2. The steady current should have at this time a value equal to A. But, as before mentioned, the actual current must be zero at the time of closing the switch and so something must happen to bring the actual current to the proper magnitude and phase to fit the steady state. This is the function of the transient term. The actual current which flows when the switch is closed at time T_2 is shown by I, the transient damped term by I', while i represents the steady current. The damped current I', which represents the free vibration of the system, will

have a period not depending upon that of the impressed force but upon the constants of the circuit. Its amplitude depends upon the constants of the circuit and the time of closing the switch. It will always so act that the actual current I is soon brought into coincidence with the steady current i. If the damping of the circuit is high this may take place within a few alternations (or even in one) but if the resistance of the circuit is low the exponential term may last for perhaps a hundred alternations.*

It is the purpose of this experiment to investigate the possible values of this steady current, so the transient term is neglected. We have

$$i = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin (\omega t - \epsilon,)$$

or, in effective values,

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

When the impressed frequency $= \frac{1}{2\pi\sqrt{LC}}$, $\omega L = \frac{1}{\omega C}$, in which case the equation reduces to $I = \frac{E}{R}$, and the circuit is said to be in resonance.

As many A. C. circuits have very low resistances, the value of the current under such conditions may become very high. As the potential difference across the terminals of the inductance and capacity depends directly upon I, it may become of such high value that the insulation of the line is broken down. In a circuit having inductance and condenser in series, the drop of potential across either the inductance or condenser may be many times larger than the E.M.F. impressed upon the circuit. In practice such a condition might arise from the capacity of a cable of low resistance and inductance of a transformer coil.

Transmission line break-downs are often attributed to resonant rise in potential due to some high current set up in it by an arcing short circuit, higher harmonics in the alternators, etc. As a transmission line consists of distributed capacity, resistance, and inductance, it is doubtful if real resonance can occur unless the resistance and inductance of the line are low

* For curves illustrating this point see Plates 3 and 4.

and some such inductance as a transformer or an alternator is attached to it. In such case the line simply acts as a condenser. A kind of resonance may occur on a transmission line if the length of the line is such that reflected waves return in the right phase to be reinforced by the impressed E.M.F. With ordinary frequencies the longest transmission line is not long enough for such an effect, so that this kind of building up of the voltage does not occur unless the alternator has a marked higher harmonic of such frequency that the above condition is fulfilled. In such a case the upper frequency current will be much accentuated and may become of great enough value to cause failures of insulators, etc.

When the inductance and capacity are connected in series as in the circuit just analyzed, the current in the line may reach

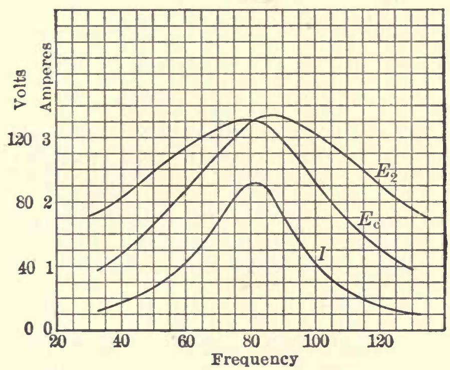


FIG. 20.

very high values and so the drop of potential across either part of the circuit may be correspondingly high.

Another type of resonant circuit is of interest; the inductance and condenser are connected in parallel and then to the supply line as shown in Fig. 21. In this case the impedance of the joint path may be very high so that only a small current flows in the line. But the inductance and condenser are in series on a local circuit and if the line frequency is such that resonance would occur for the series connection then resonance will nearly occur for this parallel circuit. The voltage drop across either the condenser or inductance cannot be greater than the voltage of the line, the resonant condition is shown by the relative current values in the line and local circuit. When resonance occurs the ammeter A_1

may read many times greater than the line ammeter. It is to be noted that in these circuits tuned for resonance the resistance in the oscillating circuit is the only thing that limits the value to which the current will build up. In the parallel circuit the value of the critical frequency (that which gives minimum wattless current in the line) is affected by the resistance in the oscillating circuit. When the charging current of the condenser is just equal to that of the inductance, we have

$$E\frac{X_c}{Z_c^2} = E\frac{X_l}{Z_l^2},$$

which gives as the resonant period of the system,

$$f = \frac{1}{2\pi}\sqrt{\frac{CR_l^2 - L}{LC\,(CR_c^2 - L)}}.$$

Connect in series an inductance, condenser and resistance and apply to the terminals of the circuit a source of variable frequency E.M.F. Calculate suitable values for L and C (from the formula given in the first part of this experiment) so that for a value of frequency at about the middle value of the obtainable range

$$f = \frac{1}{2\,\pi\,\sqrt{LC}}.$$

Connect an ammeter in series with the circuit and across the condenser connect a static voltmeter. Use an ordinary voltmeter to keep the impressed voltage constant and to measure the voltage across the inductance. With a fairly high value of resistance in the circuit vary the frequency (in about 10 steps) through the range obtainable, reading volts across the condenser and inductance and the current. Take a similar set of readings using a resistance of about one-fourth the previous value (providing the apparatus used will stand such a low resistance).

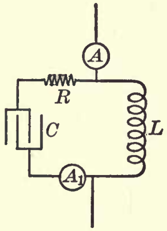


Fig. 21.

Make similar runs for the parallel circuit depicted in Fig. 21.

Plot curves between values of current, inductance drop and condenser drop against frequency as abscissæ for the series connection test and for the parallel connection, curves of line current and local circuit current, using

the same scale of abscissæ as for the first set of curves. The curves should have about the same appearance as those given in Fig. 20, which represent conditions of current and voltage for the series circuit with low value of R. For larger values of R the resonance is not so marked.

EXPERIMENT XI.

MAGNETIZATION CURVE (NO-LOAD SATURATION CURVE) OF AN ALTERNATOR AND EXTERNAL CHARACTERISTIC ON VARIOUS POWER FACTORS.

THE magnetization curve, showing the relation between voltage generated by the armature and field current, is useful to the designer, showing to what extent the field of the machine is saturated (thus giving an idea as to whether too much iron or not enough has been used on the design of the magnetic circuit), and is also valuable as being one of the curves from which the external characteristic of the machine is to be predetermined.

For constant speed (under which condition all of the above curves are to be taken) the generated voltage is directly proportional to the flux threading the armature. This being the case, a curve showing the relation between armature E.M.F. and field current really shows the relation between the field current and flux through armature, i.e., gives the saturation curve of the magnetic circuit. Not all of the flux generated by the field current cuts the armature conductors. A part of it leaks across from one pole to the adjacent pole without going through the armature core. This is called the leakage flux, and the factor by which the total flux must be divided to give the flux through the armature core is called the leakage factor. If this leakage factor remains constant with varying field excitation the armature voltage and field current actually do give the proper relation between field current and generated flux; but if the leakage factor increases with increase of voltage (likely to occur if the armature is worked at high flux density at normal voltage) then with the higher value of field current the saturation curve will give a value of flux which is increasingly lower than the actual flux through the field poles and frame. It will, however, give correctly the relation between field current and flux in armature core.

For efficient design, so far as the iron used in construction is concerned, the normal operating field current should give a

point on the saturation curve somewhat above the knee, but such a density in the armature will give iron losses too large and necessitates too high a value of field current for proper efficiency of operation. A suitable density is one which operates the magnetic circuit as a whole slightly below the knee of the magnetization curve.

Excite the alternator field by potentiometer connection to the laboratory D.C. power. With rated speed determine value of field current to give different armature voltages (the machine being unloaded), from no excitation to such as will give 25 per cent over normal voltage. Read meters at about 10 points, taking the readings closer together where the saturation curve begins to bend over. Obtain the curve for increasing values of field current only as this is the curve to be used in predetermination of regulation. Observe usual precaution in varying field current.

In obtaining the first external characteristic (as $\phi = 1$) use a lamp bank or water rheostat for load, and, with alternator running at rated speed, adjust the field current until rated voltage is obtained with full-load current flowing through armature; read terminal volts, load current, and field current. Keep field current at this value throughout test. Decrease load until about $\frac{3}{4}$ rated current is flowing and read armature current and terminal volts. Take similar readings for $\frac{1}{2}$, $\frac{1}{4}$, 0 and $1\frac{1}{4}$ of full-load current.

For power factors other than one, a synchronous motor, driving a loaded D.C. generator, is to be used as a load. Depending upon the value of field current supplied, a synchronous motor will draw either a leading or lagging current from its supply line. After the synchronous motor has been connected to the alternator, adjust the motor load until nearly full-load current is flowing from alternator. Then decrease the field current of the synchronous motor and adjust its load and also the field current of the alternator until the terminal voltage of the alternator is at its rated value, it is carrying full-load current and its actual load is 0.8 its apparent load (i.e., cos $\phi = 0.8$, lagging).

Read terminal volts, load current, field current and watts of alternator. Keep alternator field current constant at this value. Decrease the load on synchronous motor until about $\frac{3}{4}$ full-load current is flowing from alternator, readjust field current

of synchronous motor until $\cos \phi = 0.8$ and take same set of readings as before obtained for full-load current. Take similar readings for $\frac{1}{2}$, $\frac{1}{4}$ and $1\frac{1}{4}$ loads. (Zero load cannot be obtained when synchronous motor is used for load.) With full-load current on alternator and $\cos \phi = 0.8$ (leading current), obtained by over-exciting the synchronous motor field, obtain the external characteristic of the alternator for a leading current, using about the same values of load current as before and maintaining a leading load current of power factor = 0.8. Then with the synchronous motor disconnected, obtain reading of terminal volts for zero load points on the two load curves of power factor = .8.

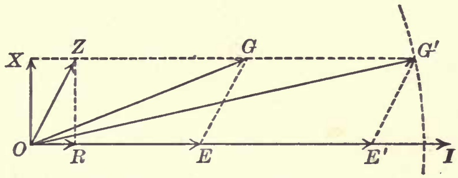
If a power-factor meter of suitable capacity is available the adjustment of the power factor of the load may be accomplished by readings of this meter instead of by computing the ratio of actual watts to apparent watts as described above.

Calculate the regulation of the alternator for the three different loads used.

EXPERIMENT XII.

FULL-LOAD SATURATION CURVE, SHORT-CIRCUIT CURRENT (SYNCHRONOUS-IMPEDANCE CURVE) AND ARMATURE CHARACTERISTIC.

The full-load saturation curve of an alternator shows the relation between the field current and terminal voltage of machine when full-load current is flowing in the armature circuit. It will be very similar in form to the no-load saturation curve but will of course be lower than this curve, if plotted on the same curve sheet. The no-load saturation curve gives the total E.M.F. generated in the armature for different values of field current while the full-load saturation curve records, for corresponding values of field current, the vector difference between the generated E.M.F. and the impedance drop in the armature due to full-load current. If the impedance of the armature remained constant for all values of field current then the actual difference



OZ, *OG*, *OG'*,—Generated volts
O, *OE*, *OE'*,—Terminal volts

FIG. 22.

between the no-load voltage and the terminal voltage of the generator while it is carrying full-load current (for the same value of field current) would grow less with increasing values of the exciting current. This is readily seen from the vector diagram given in Fig. 22. At that value of field current which gives a terminal voltage equal to zero on the full-load saturation curve the no-load saturation curve gives a voltage nearly equal to the impedance drop in the armature, so that the difference between the two is practically the full-load impedance drop in the

armature; but at higher values of excitation the vector subtraction of the impedance drop has much less effect on the terminal voltage. The above vector diagram is constructed on the assumption that the load on the alternator, when the full-load saturation curve is taken, is noninductive, the condition under which this test is to be run. If the load were inductive the above remarks about the difference between no-load and full-load saturation curves would be true only when the phase angle of the load is less than the phase angle of the armature circuit. If the external circuit has the same power factor as the armature circuit then the difference between the two curves would be the same for all values of field current.

As a matter of fact the armature impedance decreases slightly from no excitation to full excitation and more quickly for higher values of flux density. The reason for this lies in the fact that the armature teeth form a large part of the path of the leakage flux to which the armature reactance is due. The teeth carry the normal flux of the machine as well as the leakage lines and as soon as the sum of these two fluxes becomes so large that the teeth are more or less saturated then the reluctance of the path of the leakage lines increases and so the armature reactance diminishes.

The short-circuit-current curve gives the relation between the current in the armature when short circuited and the field current. It is obtained by short circuiting the armature through an ammeter (of range about 100 per cent overload for the alternator) and gradually increasing the field current until the safe carrying capacity of the armature is reached, a series of readings being taken of armature current and field current. Under these conditions it is quite evident that all of the voltage generated in the armature is used up in overcoming the armature impedance drop. The impedance of the armature will vary with the frequency; as the data from which this curve is plotted are taken as normal frequency of the machine, it is called the **synchronous-impedance** curve.

The impedance of the armature is generally calculated from the data of this curve. For any value of the short-circuit armature current the field current is obtained; the generated voltage due to this field current is taken from the magnetization curve; this generated voltage divided by the short-circuit current is called the synchronous armature impedance.

The value of impedance so obtained is far from being the true armature impedance for several reasons. Even at the highest value of armature current permissible in this test, the armature teeth are generally far from being saturated because the field current has a low value. Also in discussing the reactions which occur in an alternator it will be shown that a lagging current demagnetizes the field, the demagnetization being the greater, the larger the lag angle. Now the armature voltage on short circuit is nearly all used up in overcoming the reactance drop, as the reactance is much the larger part of the armature impedance. Hence on short circuit the current lags almost 90° behind the generated E.M.F. and so will demagnetize the field to a considerable extent. The E.M.F. taken from the magnetization curve will be much larger than the E.M.F. actually generated in the armature when short-circuit current is flowing and so the armature impedance obtained by this method will be too large.

Another method of getting the synchronous impedance is to run the alternator as a synchronous motor and over-excite its field so that the current leads the impressed E.M.F. by nearly 90°. Under such conditions the impressed voltage and induced voltage are nearly 180° apart and the IZ drop is obtained by subtracting the terminal volts from the induced voltage. The induced voltage is obtained from the magnetization curve of the machine for that value of field current which is required to bring about the above-stated relation between impressed and induced voltage. In this method also no account is taken of the fact that the field is demagnetized by the armature current and as the field is super-excited the teeth are more saturated than under normal conditions. As the first effect would give too high a value for Z and the second too low a value this method probably gives results as nearly accurate as any other method which might be employed.

Also the value of Z measured will vary according to the space position of the armature in respect to the phase of the current. If a coil side happens to be under a pole face at the time the current through it is a maximum, different value of Z will be obtained than if the coil side was in the open space between adjacent poles when the current was a maximum.

From the foregoing it is evident that the synchronous impedance of an alternator is a somewhat indefinite quantity and that its measurement involves some rather violent assumptions.

Many writers treat it as a fictitious quantity which may be used to predict the alternator regulation, and this is probably the most logical way of treating it.

The armature characteristic shows the relation between field current and load current, keeping the terminal voltage constant at its rated value for all loads. If the magnetic circuit is not near saturation this curve will be a straight line, otherwise (as e.g., when the no-load voltage is somewhere near the knee of the magnetization curve) the curve will be concave upward, it taking a greater increment of field current for a given increase of load current near full load than near no load. The curve gives the engineer an idea of what duty the exciters will have, how much they should be compounded, etc.

To get the full-load saturation curve a noninductive resistance, which may be varied from zero to a value equal to the rated terminal voltage of the machine divided by full-load current, is connected across the armature terminals in series with an ammeter; this resistance must be capable of carrying full-load

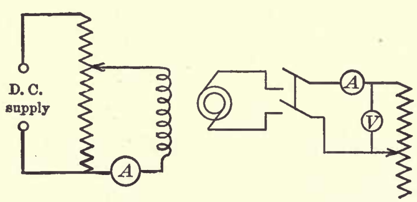


FIG. 23

current of the alternator throughout its range. A suitable voltmeter is connected across the armature terminals as shown in Fig. 23. The field is excited by potentiometer connection because the current is to be varied throughout a wide range. The same set-up of apparatus is to be used for all three curves, and speed is to be kept at rated value for all three tests. For the full-load saturation cut out all of the resistances in the armature circuit and gradually increase the field current from zero until full-load current is flowing through the armature. Read armature current, field current and terminal voltage (which will be zero for this adjustment of load). Put some resistance in the armature circuit and adjust this resistance and field current until rated current is flowing in the armature and the voltage at terminals is about $\frac{1}{8}$ of the rated value for the machine. Take

same readings as before. Again increase load resistance and field current to give about $\frac{1}{4}$ of rated voltage with full-load current flowing. Continue this until about 25 per cent above rated voltage of machine is reached.

For the short-circuit-current test **reduce the field current to zero** and then the load resistance to zero. Leaving the load resistance short circuited, increase field current until the short-circuit current is 25 per cent full-load value. Read field current and armature current. Increase field to give 50 per cent rated current in armature and again take readings. Continue until armature current is 150 per cent full-load value. The resistance of the external circuit must be reduced as low as possible for this test, large cables should be used, all connections well made, etc. Why?

To get the armature characteristic, adjust the field current to give rated voltage at no load; read value of field current and voltage of armature. Put on about $\frac{1}{4}$ load, using noninductive resistance for load, readjust field current (increasing its value only; if the proper value is exceeded, decrease the current to a small value and bring back to the desired adjustment) to give rated terminal voltage, and read field current, load current, and terminal voltage. Continue to 25 per cent overload for alternator. Make a similar run using an inductive load of $\cos \phi = 0.8$.

Plot all curves on the same sheet, using for the first curve terminal volts as ordinates and field current as abscissæ; for the second curve, volts on open circuit (to be obtained from magnetization curve and field current read in this test) as ordinates and armature current as abscissæ; for the armature characteristic use load current as abscissa and field current as ordinates.

Could the armature characteristic for load of $\cos \phi = 1$, obtained in this test be used to determine the capacity of the exciter for the alternator, if the alternator was to be used for ordinary commercial loads, such as a central-station load?

Specify a suitable rheostat for the alternator field, assuming

Note.—It has been stated above that practically all of the armature impedance consists of its reactance component. A convenient method of testing this fact is to short circuit the armature with a low value of excitation on the field. Now vary the speed of the alternator through a wide range (do not run at more than 125 per cent of rated speed), keeping the excitation constant, and read the current circulating in the armature. Account for observed facts.

the field is to be supplied from a D.C. line of proper voltage for the given field and the load on the alternator is to be a mixed one, induction motors, lights, etc.

If the armature impedance remained constant what would be the short-circuit current of the alternator with normal excitation?

EXPERIMENT XIII.

METHODS FOR PREDETERMINING THE EXTERNAL CHARACTERISTICS OF AN ALTERNATOR.

In the previous tests upon the alternator it has been shown that the drop in terminal voltage with increase of load is due to two effects, first, the actual impedance drop caused by the resistance and reactance of the armature; second, the effect of the armature current upon the field strength. The drop in terminal voltage can, therefore, be completely accounted for by considering first the voltage drop due to impedance and secondly the change in magneto-motive force due to the armature ampere turns reacting upon the main field. If, however, the whole effect is treated either from the standpoint of E.M.F. or of M.M.F. the external characteristic may be predicted with sufficient accuracy for loads of P.F. = 1, under which condition of load the armature reaction tending to change the field strength of the alternator, is negligibly small. Where it is important that the effect be calculated more closely than either the E.M.F. method or M.M.F. method permits, then a more complicated method of treatment is adopted in which each of the components of the fictitious "synchronous impedance" is dealt with separately.

The value of these methods of predicting the regulation of an alternator is realized when it is remembered that the regulation of a machine is one of its most important characteristics and that it is practically impossible to actually load the larger machines with a load of any desired power factor. Also the energy used makes the actual loading of the machine an expensive method.

The E.M.F. method of predicting the regulation gives results which are somewhat worse than the machine will actually give and so is termed the **pessimistic** method, while the M.M.F. method gives too close a regulation and so is called **optimistic.** For either method the two curves required are the short-circuit-current curve and the magnetization curve.

The E.M.F. method will first be described as it is the most important; it illustrates the application of the vector diagram for

solving problems in A.C. work and exactly the same method may be used for predicting the regulation of transformers, transmission lines, etc., as is used here for the alternator. The fictitious "synchronous-impedance drop" is considered as though made up of two components, one in phase with the current and one lagging 90° behind it. The synchronous impedance is calculated as given in Experiment 12. The ohmic resistance of the armature being known by measurement with direct current, the synchronous reactance is calculated by the formula, $X = \sqrt{Z^2 - R^2}$.

Then to predict the regulation for a load of power factor = 1, the operation is as shown in Fig. 24. The phase of the load current is assumed as OI and the rated full-load voltage, OE_t, is laid off along OI to a suitable scale. The full-load resistance drop is laid off to the same scale and is shown in Fig. 24 by OR. With a radius equal to the value of the full-load synchronous

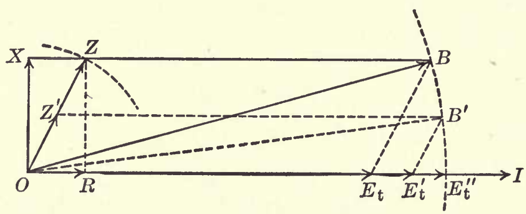


FIG. 24.

impedance in volts, a circular arc is drawn as indicated by the dotted line. From R a line is erected perpendicular to OI. The origin O is joined by a straight line to the intersection of this perpendicular and the circular arc, and this line OZ, represents the full-load impedance voltage in its proper phase with respect to the load current. The vector addition of OE_t and OZ gives the vector OB which represents the generated voltage at full load and which, in this prediction method, is assumed to remain constant for all loads. With O as center and OB as radius, a circular arc is constructed as shown, and this curve is the locus of the generated voltage as the load varies. To predict the terminal voltage at half load, take $OZ' = \frac{OZ}{2}$, (because at half load the armature-impedance drop is half as great as it is for full load), construct $Z'B'$ parallel to OI, draw $B'E_t'$ parallel to OZ' and the half-load terminal voltage is given

by the vector OE_t'. At no load the terminal voltage is OE_t''; for any other load the same construction is carried out, using an impedance drop proportional to the load assumed. The characteristic constructed from points obtained from the vector diagram will lie above that actually measured in Experiment 11.

The regulation for $\cos \phi = 1$, is given by the quotient $(OB - OE_t) \div OE_t$. The vector construction makes plain the process of predicting the regulation but will not generally be accurate enough, so that the problem is solved analytically. By reference to the vector diagram, it is seen that

$$\text{Regulation} = \left(\sqrt{(OE_t + IR)^2 + (IX)^2} - OE_t\right) \div OE_t.$$

If it is desired to predict the regulation for a load having power factor other than one the method of procedure is indicated by Fig. 25. Here a load of $\cos \phi = 0.866$ ($\phi = 30°$) is assumed.

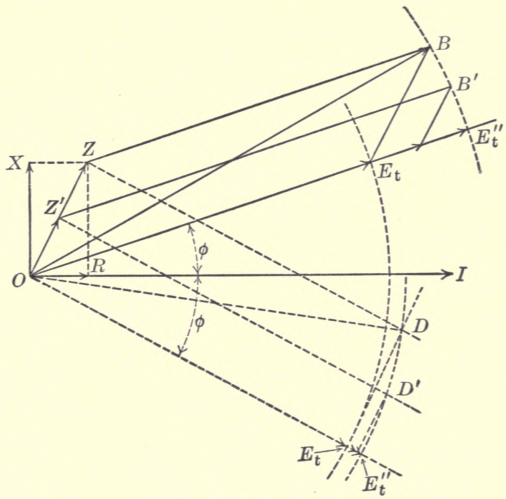


FIG. 25.

As before, the phase of the load current is assumed as OI. To predict the external characteristic for a lagging current the construction is as indicated by the full lines in Fig. 25. For a leading current of the same power factor the construction is as shown by the dotted-line construction. Because of the similarity of the construction to that used for a noninductive load, no further description is regarded as necessary.

Expressed analytically in this case the regulation is given by

$$\sqrt{(OE_t\cos\phi + IR)^2 + (OE_t\sin\phi + IX)^2} - OE_t \div OE_t$$

It will, of course, be found that the lagging-load current gives a much higher value of regulation than when $\cos\phi = 1$.

If the regulation is desired for a leading current (a condition seldom found in practice) then the line OE_t is laid off below the line representing current, and making the proper angle with the current. The construction is otherwise the same; solved analytically the regulation is given by

$$\sqrt{(OE_t\cos\phi + IR)^2 + (OE_t\sin\phi - IX)^2} - OE_t \div OE_t,$$

and if the load current is assumed to lead more than a few degrees the regulation becomes negative, i.e., the full-load terminal voltage is greater than the no-load voltage for the same value of field current.

The entire external characteristic may be calculated from the above formulæ by using for I, instead of full-load current, that current for which a point is desired on the external characteristic.

In the M.M.F. method the magneto-motive forces, corresponding to the various E.M.F.'s, are combined vectorially. The magneto-motive force for a given coil is proportional to the current through it and so the current through a coil may be used as a measure of its M.M.F. If the magnetization curve of the alternator was a straight line, the E.M.F. would be directly proportional to the M.M.F. and the two methods of prediction would give identical results.

In the M.M.F. method, the M.M.F. necessary to overcome "synchronous impedance drop" is combined vectorially with the M.M.F. necessary to give the terminal voltage and the vector sum gives the total M.M.F. which will all be effective in producing E.M.F. when there is no load on the alternator, i.e., no "synchronous-impedance drop" to overcome.

The M.M.F. necessary to overcome the impedance drop should really be considered in two parts, that to balance the IR drop and that to balance the IX drop. The procedure in this method will be understood by reference to Fig. 26. The phase of the current is given by the line OA and the M.M.F. necessary to produce full-load reactance drop is obtained from the magnetization curve and is then plotted as OM_x. The M.M.F. necessary

to produce terminal voltage plus IR drop is plotted as OM_t and the total M.M.F. is their resultant OM'_x. By reference to the magnetization curve the total induced E.M.F. (i.e., open-circuit voltage) due to M.M.F., OM'_x, can be found and so the regulation calculated. This vector problem may be solved analytically

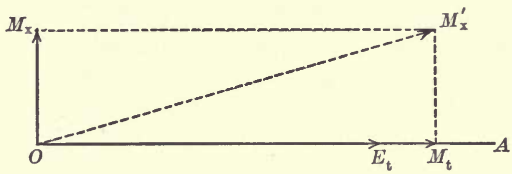


FIG. 26.

in the same manner as was the E.M.F. diagram, also different power-factor loads may be dealt with in the same way.

Both of these methods, it will be noticed, treat the armature resistance as purely ohmic, and treat the out-of-phase component of the synchronous impedance as due entirely to reactance. As a matter of fact the effective resistance of the armature is somewhat higher than the ohmic resistance due to the "load losses" (see Experiment 15) and a large part of the out-of-phase component of the synchronous impedance is not reactance, but represents the demagnetizing action upon the main field of the armature ampere turns.

A method is here given which considers all three effects which cause a change in the terminal voltage with change of load. These effects are the IR drop in the armature (R being the effective resistance), the IX drop, X being measured with normal excitation in the field, and the armature reaction.

With such field excitation as gives rated terminal voltage on the alternator at full load, and the armature stationary, impress upon the armature an E.M.F. (from some outside source) of sufficient magnitude to force full-load current through the armature. The frequency of this impressed E.M.F. is to be that at which the alternator is rated. Read current, volts and watts input to the armature. The reluctance of the path taken by the armature reactance flux varies with the angular position of the armature, as described in Experiment 15, hence these readings must be taken for several different angular positions of the armature. Take readings at six positions 30° (electrical) apart, varying the impressed E.M.F. to give full-load current for each

position of the armature. These readings are averaged and the average value of watts and voltage used to calculate Z and R of the armature. Dividing the average watts by rated current gives full-load IR drop and this is plotted in phase with the current as shown in Fig. 27. With a radius equal to the average impedance voltage (from above test) a circular arc is constructed.

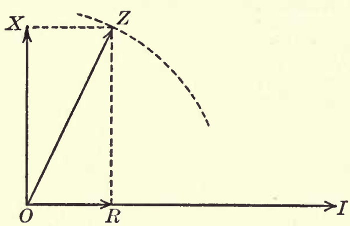


FIG. 27.

A perpendicular dropped from R to this arc intersects at Z, and OZ is then the full-load impedance drop, plotted in its proper phase relation with respect to the current. Of course OX gives the true armature full-load reactance drop. The normal values of impedance, resistance and reactance obtained from this test will be called Z, R and X.

Now with about $\frac{1}{6}$-normal excitation obtain in the same way other values for these three quantities and call them Z', R' and X'. These values will be used in connection with the synchronous-impedance test to calculate the demagnetizing effect of the armature.

Now using the value of full-load synchronous impedance as determined in Experiment 12, the effect of armature demagnetization is obtained by vector diagram as shown in Fig. 28 The full-load resistance drop is plotted as OR' and the full-load reactance drop as OX'. These values of R' and X' are for the low excitation. OZ' is full-load impedance drop. Inscribe an arc with radius equal to full-load synchronous-impedance drop and continue $R'Z'$ to intersect this arc at V. Construct VY parallel to $Z'O$ and VY is then the maximum value of the armature M.M.F. measured in terms of E.M.F. at low saturation of the field.

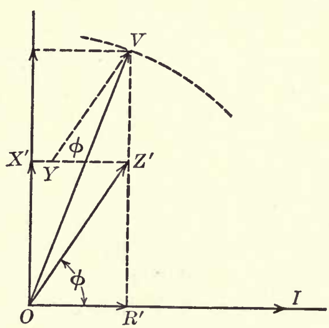


FIG. 28.

Although it is difficult to obtain a mathematically exact formula for the armature demagnetizing effect the following approximate formula may be logically deduced, $D = D_m \sin \phi$,

where D_m represents the maximum value of the effect and ϕ is the phase difference between the armature current and generated E.M.F. and D is the actual demagnetizing action. We have, therefore, $D_m = \frac{D}{\sin \phi}\cdot$ The proof of this relation may be obtained by making a simple assumption, then analyzing the integral effect of the armature currents on the field. The assumption to be made is this — that any distributed phase winding can be represented closely by a concentrated winding having the same number of ampere turns. For instance, a three-coil per phase winding of one turn per coil, carrying 100 amperes, shall be represented in our discussion by the middle coil only, and the middle coil shall be supposed to carry 300 amperes. Such a redistribution of the armature coils would, of course, mean much larger reactance per winding, but we are interested in this discussion only in the M.M.F. produced by the armature ampere turns and the above assumption is not far from the actual fact; the flux produced by the two windings would be nearly the same.

So that we shall suppose on the armature of a single-phase machine only one coil; a three-phase machine will have three

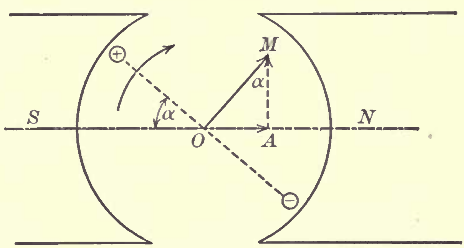


FIG. 29.

coils, etc. Considering a bipolar single-phase machine as shown in Fig. 29 and designating by ϕ the angle between the current and generated E.M.F. in the coil, we have:

$$i = I_m \cos(\alpha - \phi), \quad \text{where} \quad \alpha = \omega t,$$

therefore, the M.M.F. of the coil,

$$OM = KI_m \cos(\alpha - \phi).$$

By resolving this M.M.F. in directions parallel and perpendicular to the axis of the main field we get the instantaneous

values of the demagnetizing and cross-magnetizing effects of the armature currents.

Demagnetizing action $= OA = KI_m \cos(\alpha - \phi) \sin \alpha;$
Cross-magnetizing action $= AM = KI_m \cos(\alpha - \phi) \cos \alpha.$

To get the complete effect of these actions they must be integrated throughout a cycle.

$$\text{Demagnetization} = \text{M.M.F.}' = KI_m \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\alpha - \phi) \sin \alpha \, d\alpha.$$

$$\text{Cross-magnetization} = \text{M.M.F.}'' = KI_m \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\alpha - \phi) \cos \alpha \, d\alpha.$$

$$\text{Average value of M.M.F.}' = \frac{KI_m}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(\alpha - \phi) \sin \alpha \, d\alpha$$

$$= \frac{KI_m}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \alpha \sin \alpha \cos \phi \, d\alpha + \frac{KI_m}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 \alpha \sin \phi \, d\alpha$$

$$= \frac{KI_m}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \alpha \sin \alpha \cos \phi \, d\alpha + \frac{KI_m}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin \phi \left(\frac{1}{2} - \frac{\cos 2\alpha}{2}\right) d\alpha$$

$$= \frac{KI_m}{\pi} \left\{ \Big[\sin^2 \alpha \cos \phi\Big]_{-\frac{\pi}{2}}^{0} + \Big[\sin^2 \alpha \cos \phi\Big]_{0}^{\frac{\pi}{2}} + \left[\frac{\alpha \sin \phi}{2}\right]_{-\frac{\pi}{2}}^{0} + \right.$$

$$\left. \left[\frac{\alpha \sin \phi}{2}\right]_{0}^{\frac{\pi}{2}} - \left[\frac{\sin 2\alpha \sin \phi}{4}\right]_{-\frac{\pi}{2}}^{0} - \left[\frac{\sin 2\alpha \sin \phi}{4}\right]_{0}^{\frac{\pi}{2}} \right\}$$

$$= \frac{KI_m}{\pi} \Big\{ [-\cos \phi + \cos \phi] + \qquad (A)$$

$$\left[\frac{\pi \sin \phi}{4} + \frac{\pi \sin \phi}{4}\right] + [0 - 0] \Big\}$$

$$= \frac{KI_m}{\pi} \times \frac{\pi \sin \phi}{2} = \tfrac{1}{2} KI_m \sin \phi.$$

In the single-phase alternator, therefore, there does exist an effective demagnetizing action and this action is directly proportional to $\sin \phi$, where ϕ is the phase displacement of generated E.M.F. and the current I in the coil.

It is also interesting to notice the meaning of the expression marked (A); while the coil passes from position $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ this term changes its sign from $-$ to $+$. In the single-phase alternator there is, therefore, the pulsating M.M.F. which tends first to weaken and then to strengthen the main field. This action of the armature produces eddy-current losses in the pole faces

of the machine but does not affect the average strength of the alternator field to a noticeable degree.*

The effective value of the cross-magnetization in the single-phase machine may be obtained in a manner similar to that used for M.M.F.′ We have

$$\begin{aligned}
\text{M.M.F.}'' &= \frac{KI_m}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos^2\alpha\cos\phi - \cos\alpha\sin\alpha\sin\phi)\,d\alpha \\
&= \frac{KI_m}{\pi}\left[\sin^2\alpha\sin\phi + \frac{\alpha\cos\phi}{2} + \frac{\cos 2\,\alpha\cos\phi}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
&= \tfrac{1}{2}\,KI_m\cos\phi.
\end{aligned}$$

In the case of the three-phase machine we must replace the actual distributed phase windings by three " equivalent " coils 120° apart. Using part of the previous expansion we may immediately write down for the demagnetizing action of a three-phase armature, carrying equal currents in the three phases.

$$\begin{aligned}
\text{M.M.F.}' = \frac{KI_m}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Big\{ &[\cos\alpha\sin\alpha\cos\phi + \sin^2\alpha\sin\phi] + \\
&\left[\cos\left(\alpha+\frac{2\,\pi}{3}\right)\sin\left(\alpha+\frac{2\,\pi}{3}\right)\cos\phi + \sin^2\left(\alpha+\frac{2\,\pi}{3}\right)\sin\phi\right] + \\
&\left[\cos\left(\alpha+\frac{4\,\pi}{3}\right)\sin\left(\alpha+\frac{4\,\pi}{3}\right)\cos\phi + \sin^2\left(\alpha+\frac{4\,\pi}{3}\right)\sin\phi\right]\Big\}\,d\alpha
\end{aligned}$$

$$\begin{aligned}
= \frac{KI_m}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\phi \Big\{ &\cos\alpha\sin\alpha + \cos\left(\alpha + \frac{2\,\pi}{3}\right)\sin\left(\alpha + \frac{2\,\pi}{3}\right) + \\
&\cos\left(\alpha+\frac{4\,\pi}{3}\right)\sin\left(\alpha+\frac{4\,\pi}{3}\right)\Big\}\,d\alpha
\end{aligned}$$

$$+\frac{KI_m}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\phi\left\{\sin^2\alpha + \sin^2\left(\alpha+\frac{2\,\pi}{3}\right) + \sin^2\left(\alpha+\frac{4\,\pi}{3}\right)\right\}d\alpha.$$

Expanding the expression in the bracket of the first integral we get

$$\cos\alpha\sin\alpha \qquad = \cos\alpha\sin\alpha$$

$$\begin{aligned}
\cos\left(\alpha+\frac{2\,\pi}{3}\right)\sin\left(\alpha+\frac{2\,\pi}{3}\right) = &\left\{\cos\alpha\left(-\frac{1}{2}\right) - \sin\alpha\left(\frac{\sqrt{3}}{2}\right)\right\} \\
&\times\left\{\left(-\frac{1}{2}\right)\sin\alpha + \left(\frac{\sqrt{3}}{2}\right)\cos\alpha\right\}
\end{aligned}$$

* For experimental proof of the presence of this pulsating armature reaction, see Appendix, Plate 5. Plates 16, 17, and 19 also show the same effect.

$$= \frac{\cos\alpha\sin\alpha}{4} - \frac{\sqrt{3}\cos^2\alpha}{4} + \frac{\sqrt{3}\sin^2\alpha}{4} - \frac{3\sin\alpha\cos\alpha}{4}.$$

$$\cos\left(\alpha+\frac{4\pi}{3}\right)\sin\left(\alpha+\frac{4\pi}{3}\right) = \left\{\left(-\frac{1}{2}\right)\cos\alpha - \left(-\frac{\sqrt{3}}{2}\right)\sin\alpha\right\}$$
$$\times\left\{\left(-\frac{1}{2}\right)\sin\alpha + \left(-\frac{\sqrt{3}}{2}\right)\cos\alpha\right\}$$
$$= \frac{\cos\alpha\sin\alpha}{4} + \frac{\sqrt{3}\cos^2\alpha}{4} - \frac{\sqrt{3}\cos^2\alpha}{4} - \frac{3\sin\alpha\cos\alpha}{4}.$$

The sum of the right hand members of the last three equations is zero. By comparison with the form of expression obtained for the single-phase alternator we see that the first integral in the above expression for demagnetizing action is that one which, in the single-phase machine, represented the pulsating component of the demagnetizing action. As this integral does not exist in the three-phase machine (owing to the fact that the above sum reduces to zero) we conclude that there is no pulsating armature reaction in the three-phase machine.

The last integral must still be evaluated and we do this by first expanding the various terms.

$$\sin^2\alpha = \frac{1}{2} - \frac{\cos 2\alpha}{2},$$

$$\sin^2\left(\alpha+\frac{2\pi}{3}\right) = \frac{1}{2} - \frac{\cos 2\left(\alpha+\frac{2\pi}{3}\right)}{2},$$

$$\sin^2\left(\alpha+\frac{4\pi}{3}\right) = \frac{1}{2} - \frac{\cos 2\left(\alpha+\frac{4\pi}{3}\right)}{2},$$

so that

$$\frac{KI_m}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin\phi\left\{\sin^2\alpha + \sin^2\left(\alpha+\frac{2\pi}{3}\right) + \sin^2\left(\alpha+\frac{4\pi}{3}\right)\right\}d\alpha$$

$$= \frac{KI_m}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{3}{2}\sin\phi\,d\alpha -$$

$$\frac{KI_m}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left\{\frac{\cos 2\alpha}{2} + \frac{\cos 2\left(\alpha+\frac{2\pi}{3}\right)}{2} + \frac{\cos 2\left(\alpha+\frac{4\pi}{3}\right)}{2}\right\}d\alpha.$$

$$\text{M.M.F.}' = \text{Demag. action} = \frac{KI_m}{\pi}\times\frac{3\pi}{2}\sin\phi = \frac{3}{2}I_m K\sin\phi.$$

By an exactly similar process we may obtain the cross-magnetization and shall find that M.M.F.″ $= \frac{3}{2} KI_m \cos \phi$.

The absence of the pulsating term in the balanced polyphase machine might have been surmised when it is remembered that the polyphase currents on the armature produce a uniform magnetic field which revolves backward on the armature at synchronous speed, so that whatever effect it produces must be stationary in space because the armature is turning forward at synchronous speed.

Now in Fig. 29 it is evident that the value of the demagnetizing action in the synchronous-impedance test is $Z'V$, and that

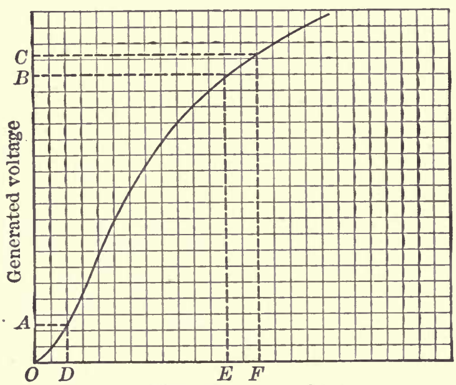


FIG. 30.

the angle between generated E.M.F. and current is ϕ. Hence the maximum value of the armature M.M.F. (in terms of volts at low saturation of field) is equal to VY because $VY = \dfrac{Z'V}{\sin \phi}$.

This M.M.F. is now to be changed to its equivalent in volts at **normal saturation** of the alternator field. Referring to Fig. 30 (which gives the magnetization curve of the alternator) we have

OA = D (max. demag. action in volts at low saturation).
OD = equivalent M.M.F.
OE = normal excitation of field.
EF = OD.

Then BC = max. demag. action in volts at normal saturation of field.

The vector diagram for external characteristic prediction is now constructed as shown in Figs. 31, 32, 33, which are for $PF = 1$, $PF = 0.8$ (leading) $PF = 0.8$ (lagging) respectively.

In these figures

OI = phase of load current.

OR = full-load IR drop (effective resistance at normal saturation).

OX = full-load IX drop at normal saturation.

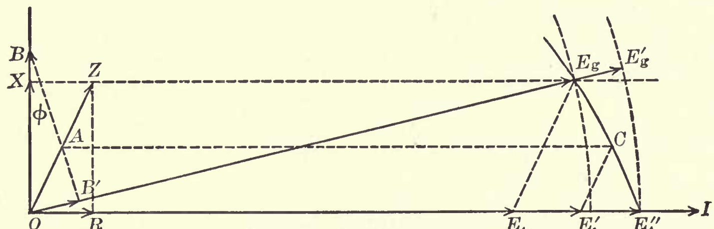
OB = maximum demagnetizing effect in volts = BC of Fig. 30.

OE_t = rated full-load voltage.

$OBB' = E_gOI = \phi$ = phase difference between generated voltage and armature current at full-load.

$E_gE_g' = OB' = OB \sin \phi$ = armature demagnetizing effect at full load.

At full load the generated voltage is OE_g, but at no load (the field current remaining the same) the generated voltage = OE_g',



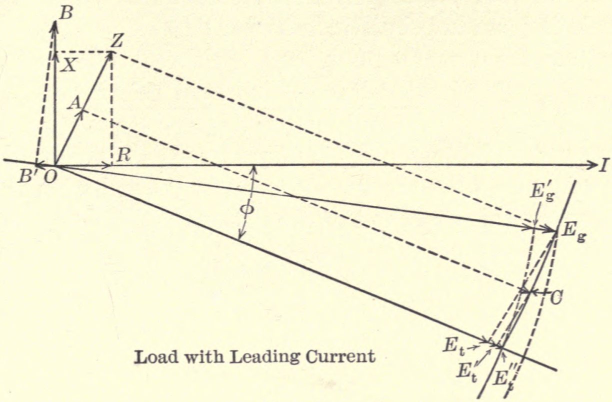
Non-inductive load

FIG. 31.

because E_gE_g', the amount of voltage neutralized at full load by the armature reaction, is not neutralized at no load, and so appears as generated voltage.

As the load decreases, the armature demagnetizing action, OB, which is directly proportional to the armature current, decreases and the angle ϕ also decreases. These combined effects give for the locus of the generated voltage, not a circular arc through E_g, as was assumed in the other two methods of prediction, but the curve E_gE_t''. The student may actually construct this locus or it may be closely approximated by first drawing through E_g', an arc with O as center. This arc intersects the line assumed as the phase of the terminal voltage at E_t''. By joining E_g to E_t'' with a curve of the form shown in the different figures the locus is obtained very closely without the trouble of so much construction as is necessary to get the

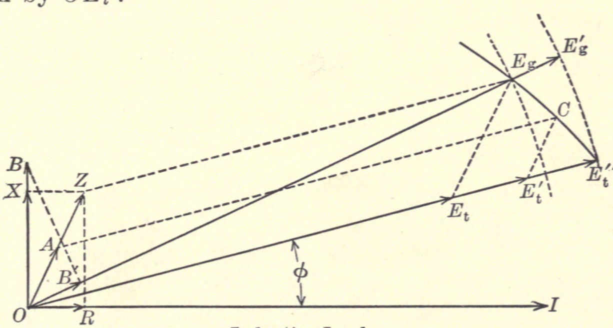
exact locus. Having thus obtained the locus of the generated voltage, the method of predicting the voltage for any load is the



Load with Leading Current

FIG. 32.

same as for the E.M.F. or M.M.F. method. To get the terminal voltage at ½ load, e.g., take $OA = \frac{OZ}{2}$, draw AC parallel to OE_t, draw CE_t' parallel to AO and the terminal voltage for half-load is given by OE_t'.



Inductive Load

FIG. 33.

Using the same alternator as was tested in Experiment 11, and 12, make all necessary measurements to predict, by the three methods given in this analysis, the external characteristic. Plot the entire characteristic by the last method, for loads of the same power factors as used in Experiment 11. Indicate on the curves so constructed the points actually obtained in Experiment 11.

On another sheet plot the entire characteristic by the E.M.F. and M.M.F. method for the same power factors. By comparison with the results of Experiment 11, see whether these methods are pessimistic and optimistic as stated?

Calculate from results obtained by these two methods the regulation, i.e. $\left(\dfrac{\text{no-load voltage} - \text{full-load voltage}}{\text{full-load voltage}}\right)$, for loads of all three power factors used in Experiment 11. How do these results compare with the actual test results and with those obtained by the third method of prediction?

EXPERIMENT XIV.

EFFICIENCY OF AN ALTERNATOR BY RATED MOTOR.

The most direct method for measuring the efficiency of an alternator (or any generator) would be to actually load the machine, obtain the output by reading the electrical instruments in the load circuit and measure the mechanical input by means of a cradle or transmission dynamometer. Of course, this method for getting the efficiency of an alternator is only to be used with small machines; with large machines the cost of the power consumed while conducting the tests becomes important. Also the question of proper loading facilities for large machines brings up difficulties from the electrical side of the test and the measurement of the power input by dynamometers, etc., presents mechanical difficulties. So for testing the efficiency of large machines one of the various "loss" methods is used.

For small machines, however, the input-output method is simple and direct; but instead of using a dynamometer to measure the input we may drive the alternator by a motor, and, when the various motor and transmission losses are known, the alternator efficiency can be calculated by reading the electrical output of the alternator and electrical input to the driving motor for various loads. This is termed the "rated motor" method.

When a certain load is put upon the alternator the input to the driving motor will be larger than the alternator output, due to three general losses which occur in the combination of machines — alternator losses, belt-transmission losses (this factor will be zero if direct coupling of machines is employed) and motor losses. From this it is evident that if the motor input is read and motor losses and transmission losses are subtracted from this reading then the difference will be the input to the alternator. By this method the efficiency formula becomes:

$$\text{Efficiency} = \frac{\text{alternator output}}{\text{motor input} - (\text{motor losses} + \text{transmission losses})} + \text{alternator-field loss.}$$

The losses in belt transmission and their variation with load are not easily separated from the alternator losses and they will, therefore, be assumed constant and equal to (A)* per cent of the alternator input at no load.

The motor losses consist of field I^2R loss, independent of load, armature I^2R loss varying with load, and stray-power losses, involving eddy-current losses and hysteresis in the armature core and pole faces, bearing and brush friction, windage, etc. These losses, while they probably do change somewhat with the load, will be reckoned constant. Hence the only variable loss to consider in the motor is its armature I^2R loss. If the motor used has copper brushes, the armature resistance may be treated as constant, but when carbon brushes are used the resistance of the armature circuit changes very much with the current flowing through it. The resistance decreases with increase of current, so that if the armature resistance were measured at a small value of current and this resistance were used to calculate the full-load I^2R loss in the armature, much too high a loss would be obtained. The resistance of the armature must, therefore, be measured with various values of current from small values to 125 per cent of full-load current of the motor. From this data a resistance-current curve is constructed and in calculating the I^2R loss for any particular value of current, the proper value of R must be obtained from the curve.

It has been stated that the hysteresis and eddy-current losses in the motor (for a constant motor speed) may be treated as a constant. This statement, as well as the one regarding a constant I^2R loss in the field, is correct because during the test the motor field current is to be maintained constant.

But the alternator must be run at its rated speed for all loads and the motor would naturally slow down as its load increases. The motor speed would ordinarily be adjusted by weakening its field current, but the field current is to be kept constant. Hence the armature circuit of the motor must contain a low, variable, resistance which can be cut out as the motor load increases, so that the alternator speed can be held at a constant value. This resistance should give, at light-load current, a voltage drop equal

* The factor A varies so with different belting that no definite value can be assigned, but the instructor must use his judgment for individual installations.

to about 10 per cent of the rated voltage of the motor and must be capable of carrying for a short time such motor current as will run the alternator at about 25 per cent overload.

With connections as designated in Fig. 34, and the alternator running light (field excited) adjust the resistance in series with motor armature until it uses up about 10 per cent of the impressed voltage of the motor. Then adjust the motor field rheostat until the alternator is running at its rated speed and keep the motor field current at this value throughout the test. With the alternator running at rated speed get motor speed.

Take off belt (or coupling), run motor at this speed, having its field current at the value just determined, read input to motor

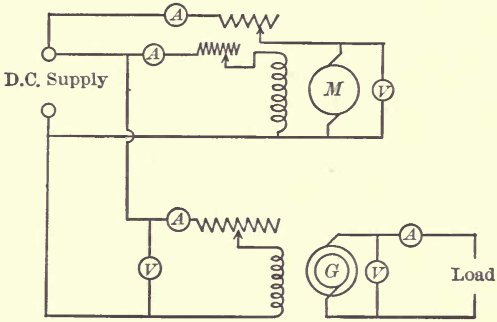


FIG. 34.

armature. These readings are to be used in calculating the alternator input, as indicated below. After these readings have been obtained the alternator is again connected to its driving motor.

Keeping the alternator terminal voltage and its speed constant, vary its load (use noninductive load) in steps so as to get 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 and $1\frac{1}{4}$ times its rated load. For each adjustment of load, read volts and amperes alternator load, amperes in alternator field and voltage of line supplying the field current, motor field current, motor armature amperes and volts and alternator speed. By " fall of potential " method obtain the resistance of the motor armature for several values of current covering the range of current used by the motor during the test.

If $W' =$ watts input to motor, alternator disconnected.

$I_1 =$ amperes input to armature of motor, alternator disconnected.

W'' = watts input to motor at some particular alternator load.

I_2 = amperes in motor armature alternator at the same load.

R_a' = resistance of motor armature, alternator disconnected.

R_a'' = resistance of motor armature, alternator loaded.

Then the alternator input at this particular load will be

$$W'' - I_2^2 R_a'' - (W' - I_1^2 R_a').$$

Draw armature resistance curve using current as abscissa. Calculate alternator efficiency for the loads used above and construct efficiency curve through the points obtained.

EXPERIMENT XV.

EFFICIENCY OF AN ALTERNATOR BY THE "LOSS" METHOD.

THE efficiency of any machine is expressed by the formula:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}.$$

The utilization of the efficiency formula in this form necessitates the measurement of both output and input at the different loads for which the efficiency is desired. As this method is many times impracticable and expensive, the efficiency of a machine is generally determined by use of the formula:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{losses in machine}}.$$

It is seen that this is merely a different way of putting the same formula, because input = output + losses.

The use of this formula for the determination of efficiency is more convenient than the previous one; a more important reason for its use in laboratory tests, however, is that all the different losses in the machine must be separately determined. Thus the action of the machine must be fully analyzed to find out how the different losses compare in magnitude, how they vary with the load, etc. In using the first formula given, the action of the machine itself need receive no attention whatever; the only measurements involved are the two powers, input and output, and so the test is of little educational value.

In the "loss" method the various losses are plotted in the form of curves, using load current as one variable and losses as the other. From the separate loss curves, a total loss curve may be obtained. Then for a certain output, the efficiency of the machine is obtained by reading from this curve the losses for the assumed load and employing the formula:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{losses}}$$

In the alternator the different losses are field I^2R loss and field-rheostat I^2R loss; armature I^2R loss; windage, bearing and brush friction, hysteresis and eddy-current losses, all grouped under the name of "stray power;" and extra iron loss designated the "load loss." Of these losses the stray power increases slightly with the load, the field and rheostat loss increases to a slight extent, the armature I^2R loss increases with the second power of the load and the load loss increases with some power lower than the second.

As the alternator is supposed to be a constant-voltage machine, and as it has been shown in Experiment 12 that the field current must be increased to maintain constant voltage as the load increases, it is quite evident that the field loss will increase with load. If the armature characteristic of the machine under test has already been obtained the curve of field and rheostat loss is readily plotted by multiplying the values of field current by the voltage of the line supplying the excitation current. Of course the loss in the field rheostat must be charged to the alternator as it is one of the power losses necessary for the normal operation of the machine.

The stray-power losses will in part be independent of load and in part increase with the load. Air friction, bearing and brush friction are constant quantities within the limits of experimental accuracy; the iron losses, however, will slightly increase with load because the speed remains constant and the flux density increases with the load due to increase of field current. In this test the hysteresis and eddy-current loss will be considered constant and their increase will be measured as part of the load losses.

The armature I^2R loss is ordinarily obtained by multiplying the square of the load current by the armature resistance as measured by direct current. As the load on an alternator increases, so does the leakage flux which encircles the armature coils. This flux, when the coil side is in the interpolar space, passes through a short path in the armature core through the teeth of the armature and thence completes its circuit through air as illustrated by Fig. 35. As the coil side moves under a pole the pole face forms part of the path of this flux and the flux increases because the pole face offers less reluctance than the air path. This variation in the magnetic reluctance, combined with the regular alternation of the current causes hysteresis losses in

the teeth (other than those produced by the main field flux variation in the teeth). Also as these leakage lines utilize the pole face for part of their path, and as they move across the pole face due to the motion of the armature, hysteresis and eddy-current losses are caused in the pole faces.

As a matter of fact, when the coil side comes under the pole face the leakage lines do not exist in the form shown in Fig. 35, but the magneto-motive force of the coil brings about a shifting

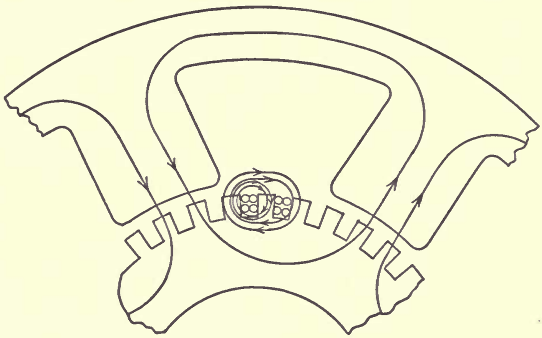


FIG. 35.

of the main field. The armature ampere-turns produce a cross-magneto-motive force, concentrating the main flux in one or the other pole tip. In polyphase alternators this cross M.M.F. is practically constant in direction and magnitude for a given load current so that in this type of machine the field is constantly distorted in one direction; in the single-phase machine, however, the effect is an alternating one, changing in magnitude and direction. It results in a surging of the field flux from one pole tip to the other, thus producing eddy currents and hysteresis losses in the pole faces. The amount of surging is limited by the eddy currents; if it is desired to keep this amount of surging very small, the paths for the eddy currents are made of low resistance so that, for a given shifting of the flux, the eddy currents are as great as possible. For the purpose of limiting this field surging in machines where it may produce bad effects (as in synchronous motors and rotary converters) heavy copper grids are imbedded in the pole face to furnish low resistance eddy-current paths.

The various iron losses which are caused by the armature current are grouped under the name "load losses." The

ordinary methods of measuring these load losses are purely empirical, and the accuracy of the results obtained is much to be doubted.

It is common practice in laboratory tests to short circuit the armature and excite the field sufficiently to cause full-load current to flow through it. The energy used up in the armature and pole faces is measured by a "rated motor." The ohmic I^2R loss in the armature is subtracted from the value obtained and the load loss is reckoned as $\frac{1}{3}$ of the remainder. The fraction $\frac{1}{3}$ is purely arbitrary. It is easy to see that the full value of the remainder could not be taken as the load loss because the test is made under a very low value of excitation, under which condition the armature reaction on the field will have much larger effect than when the field is more nearly saturated.

It is proposed in this test to modify the ordinary method of obtaining armature I^2R loss and load losses by a method in which the conditions approach actual load conditions more closely than in the test commonly employed. At the same time the increase in hysteresis and eddy-current loss due to increasing field current will be measured and included as part of the armature I^2R loss and load loss.

The method here proposed for obtaining the hysteresis and eddy-current losses (of constant value) and the armature I^2R and load losses will probably not be considered commercially important, because, although very little power is required for the test, an A.C. service is required which has the same voltage as the machine rating and a current capacity equal to full-load current of the machine; generally such service will not be at hand. The method is applicable to laboratory tests, however, and is carried out as follows:

Adjust the value of alternator field to normal value, no load. Run the machine as a synchronous motor, unloaded, and adjust the impressed voltage until the P.F. of the motor is unity. The watts input under these conditions gives the stray power + armature I^2R + a small load loss which will be included with the stray power. From the watts input under these conditions subtract the armature I^2R (ohmic) and call the remainder **stray power** which will be regarded as constant for all loads.

Now increase the field current of the machine to its proper value for $\frac{1}{4}$ load, as obtained from the armature characteristic for load of P.F. = 1 in Experiment 12. Increase the impressed

voltage until $\frac{1}{4}$-full-load current is reached in the armature (machine still running as an unloaded synchronous motor) and read the watts input. Under this condition the armature current will lag behind the impressed E.M.F. and will tend to magnetize the alternator field, so the input will be larger than it should be for P.F. = 1. To get rid of this error in the measurement leave the value of alternator field at its proper $\frac{1}{4}$-load value, and decrease the impressed E.M.F. until $\frac{1}{4}$-full-load current again flows in the armature, the current now leading the impressed E.M.F., and read the watts input to the armature. With this condition the armature current will tend to demagnetize the alternator field, making the input less than it should be. Average the two

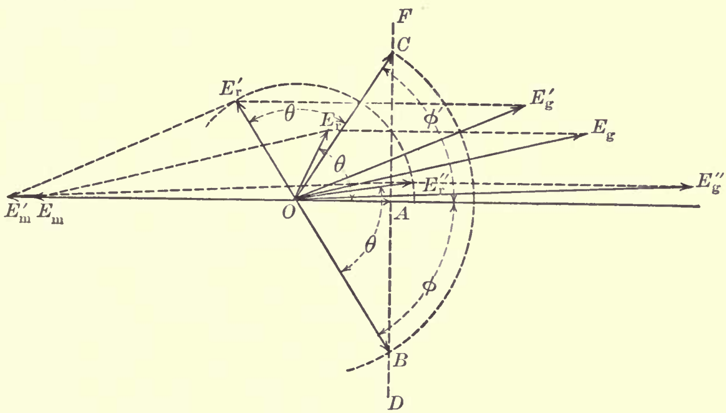


FIG. 36.

values of input obtained for $\frac{1}{4}$ load, subtract the "stray power" and the remainder will be (armature I^2R + load loss) for this value of load current.

It has been shown in a previous experiment that the armature reaction of a synchronous machine will magnetize the field if the current leads the generated E.M.F. and will demagnetize the field if the current lags behind the generated E.M.F.

That the armature current does lead the generated E.M.F. when the alternator (running as a synchronous motor) is generating an E.M.F. less than the impressed may be readily shown by vector diagram. That the current taken from the line by the machine, when the line E.M.F. is less than the generated E.M.F., lags behind the generated E.M.F. may also be shown by vector diagram.

In Fig. 36, E_m represents the generated E.M.F. in the armature of the alternator (running as a synchronous motor). The impressed E.M.F. is given by E_g and the resultant of the two by E_r. The load on the motor is constant and is supplied by an electrical input of $(E_m \times OA)$ where OA represents the current for that value of impressed E.M.F. which makes the input current a minimum. If the impedance and armature constants are known, E_r (the voltage necessary to overcome the armature impedance drop when current OA is flowing) may be plotted in proper phase and magnitude. If $Z =$ armature impedance,

and $$\tan \theta = \frac{X_a}{R_a}$$

then for any magnitude and phase of armature current the necessary impedance voltage may be plotted.

If ϕ is the angle between the current and E_m reversed in phase, it is evident that for armature current, I_a, the condition which must hold is, $E_m I_a \cos \phi =$ constant.

Hence, if in Fig. 36 the phase of E_m is fixed, the locus of the armature current must be the line FD. The value of quarter-load current is OB $(= OC)$, and the quarter-load excitation produces an E.M.F. $= E_m'$. The necessary impedance voltage is E_r' and the impressed voltage is either OE_g' or OE_g''. Then with a motor voltage of E_m', either OE_g' or OE_g'' may be impressed on the motor and $\frac{1}{4}$-load current will flow through the armature. But current OC will demagnetize the field and the current OB will magnetize the field and these two armature reactions will be equal in magnitude (as $\phi = \phi'$). Hence the input with impressed voltage $= OE_g'$ will give the proper I^2R loss for $\frac{1}{4}$ load but the core loss will be less than it should be by an amount $= X$. When the impressed voltage is OE_g'' the I^2R loss will again be correct but the core loss will be greater than it should be by approximately the same amount X. (This reasoning is on the assumption that the field and armature are not operated near the saturation point.)

Hence the average of the two inputs, when the impressed E.M.F. is OE_g' and OE_g'' will give the proper I^2R loss and the proper core loss, part of which is the load loss.

Increase the alternator field to its proper value for $\frac{1}{2}$ load. Increase the impressed E.M.F. until $\frac{1}{2}$-full-load current is flowing in the armature and read input. Then decrease the impressed E.M.F. sufficiently to give the same value current leading the

impressed E.M.F. and again read input. Calculate (armature I^2R + load loss) as before. Obtain similar readings for $\frac{3}{4}$, full, and $1\frac{1}{4}$ full load.

Measure the ohmic resistance of the armature. Calculate the armature I^2R loss, and, so by differences, obtain the load loss for the various loads.

Plot on one sheet of section paper the field circuit I^2R loss, the stray power loss, the armature I^2R, the load loss and also the total loss curve, all curves being plotted against amperes ouput as abscissa. Construct the efficiency of the alternator for $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, full and $1\frac{1}{4}$ full load at P.F. = 1. Compare with the efficiency obtained by the rated motor test. Which method is the more accurate and which is preferable for testing large machines?

EXPERIMENT XVI.

PARALLEL OPERATION OF ALTERNATORS.

The question of series or parallel operation of alternators has become important with the present practice of power generation. Nearly all large stations generate alternating current and in a single station are installed many generators. The possibility of operating the generators as separate systems, keeping one or more feeders for a certain machine, decreases with the number of generators installed; also such service is unreliable and difficult to maintain. For reliability of operation and convenience in making repairs, etc., it is best to have all machines supplying power to the same bus bars and all of the feeders connected to the same station bus.

As the service for most light and power loads is required to be of constant potential it is evident that the alternators, if operated on a common bus, must be connected in parallel. If the different machines were connected in series the bus voltage would vary with the number of alternators supplying the load.

Series operation of alternators on ordinary circuits is impossible unless the machines are rigidly coupled together, because if the load is at all unbalanced between the two machines they immediately pull out of phase, until their E.M.F.'s are acting in opposition instead of in series. The reason for this fact can be shown by constructing power curves as shown in Fig. 37. In the upper figure are given the E.M.F. waves *A* and *B*, of the two machines, their combined E.M.F. acting in series *C*, and the load current *D*, which is taken as a lagging behind the E.M.F., *C*. It is assumed that the two machines were operating with their E.M.F.'s in phase and that for some reason *B* began to lag behind *A* a little. In the lower figure are shown the power curves for the two machines, constructed by using as ordinates the product of the current *D* and the voltages *A* and *B*. It will be seen that *B*, the machine which is assumed lagging, has more load than *A*; this unequal division of the load will act to slow down *B* still more and allow *A* to speed up, making the distribution of load still more unequal,

until *B* is supplying whatever load there is and also supplying power to run *A* as a motor. This condition occurs when the two E.M.F.'s are nearly 180° out of phase, when referred to each other. As they pull more out of phase the line voltage *C* decreases until finally the voltage on the load circuit is so low that the load supplied by the two machines is practically nothing.

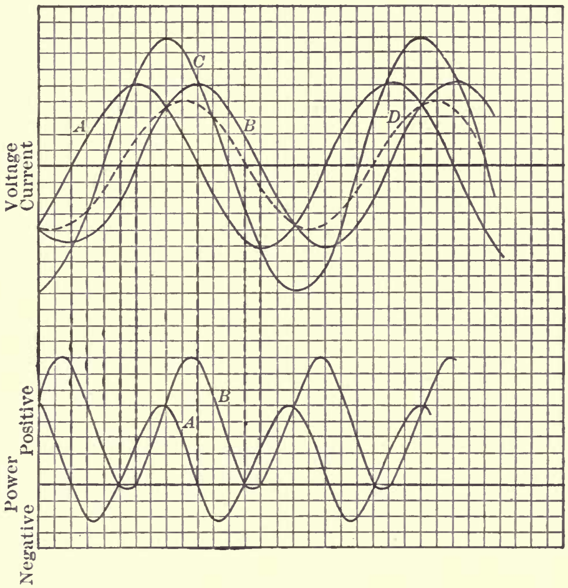


Fig. 37.

It is, therefore, evident that series operation of alternators for loads where the current lags behind the generated voltage of the machines is not feasible, the operation being a case of unstable equilibrium. As practically all commercial circuits furnish a load where the current lags to some extent, it is seen that, even if series operation of alternators was desirable, the machines could only operate successfully if mechanically coupled together. If the current *D* (Fig. 28) was in phase with the voltage *C*, the operation would not be unstable, but the machines would not tend to maintain any special phase position with respect to each other; any phase displacement between the two would bring into action no force tending to hold the machines in the same phase. Hence any tendency of one driver to get ahead of the other would not be counteracted and the relative phases of

the E.M.F.'s, *A* and *B*, would change at random, thus causing the line voltage *C* to vary between zero and a maximum equal to the sum of the voltages of the two machines.

By means of a vector diagram, the question of stability is easily investigated. In Fig. 38 are shown the three possible conditions, leading, in-phase and lagging load current. By supposing machine *A* to get ahead of *B* and then examining the distribution of load between the two it is seen that (*a*) with

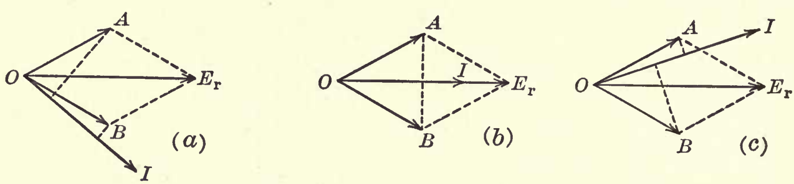


Fig. 38.

lagging-current instability exists; (*b*) with in-phase current indifferent stability exists; and (*c*) with leading current the operation is more or less stable, depending upon the angle of lead. As this case is seldom met in practice it has little commercial importance.

Machines operating in parallel are generally quite stable and satisfactory in their operation. If one machine speeds up, it immediately takes more of the load and so is held back. A complete analysis of the factors which affect the parallel operation of alternators cannot be attempted here, but only those which are to be tested in the laboratory.

When it is desired to connect another D.C. generator to the line to which power is already being supplied by other machines, the incoming machine is merely brought up to line voltage, tested for correct polarity and switched on to the line. It will then take its share of the load if its field excitation is properly increased.

To switch an incoming alternator to the line is not such a simple matter. Before the switch may be closed three conditions must be satisfied.

1. Machine voltage = line voltage.
2. Machine frequency = line frequency.
3. Phase of machine E.M.F., 180° out of phase with line E.M.F.

There is a fourth condition which must be fulfilled before the synchronizing switch may be closed, but as this condition cannot be affected by the operator it is not grouped with the other three, all of which are adjustable by manipulation of rheostats, etc. This fourth condition requires that the E.M.F. wave generated by the alternator be similar in form to that of the line E.M.F. If the wave forms are dissimilar there may be a large interchange of current between the machine in question and the other machines connected to the line, and this wattless current circulating in the local path does no useful work but helps to heat the alternators.

The cause of this circulating current may be seen by reference to Fig. 39, in which the line E.M.F. is shown by the full-line curve A, the generator E.M.F. by the dotted-line curve B and the difference between the two by the full-line curve C. The line E.M.F. is supposed to be approximately a simple sine wave while the generator is shown with a fifth harmonic. The unbalanced voltage consists principally of this fifth harmonic, shown by curve C. Now it is to be noticed that the two E.M.F.'s are such that both give the same reading on an A.C. voltmeter, i.e., they have the same effective values, and it is evident that this unbalanced voltage, shown by C, cannot be reduced to zero under any condition of field adjustment, etc. Now this voltage C will force a current to flow from the armature of the machine in question through the armature of the other machines connected to the line, in parallel. This circulating current not only produces unnecessary heating of the generators but, if of appreciable magnitude, may seriously affect the stable operation of the machine. It is likely to produce a tendency for the generator to " hunt," an effect which is explained in a later experiment.*

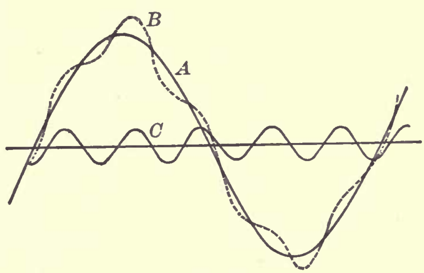


FIG. 39.

The operation of closing the switch which connects the incoming machine to the line, with the necessary adjustments of

* For illustration of circulating current between two alternators, see Appendix, Plate 6.

voltage, speed, etc., to satisfy above conditions is termed "synchronizing" the alternator.

When the second condition is approximately fulfilled (by speed measurement or otherwise) the machine voltage is made equal to the line voltage by field adjustment. Then by some sychronizing device, as lamps or synchronscope, condition No. 2 is satisfied more closely. By watching the synchronizing device, the operator can tell when the third condition is fulfilled and the switch is closed.

The connection of lamps to be used for synchronizing is given by full lines in Fig. 40. The lamps used must each be built for a voltage equal to that of the alternator (e.g., a 220-volt alternator requires two 220-volt lamps). These lamps complete the circuit consisting of the alternator armature and the armatures of the other alternators already operating on the line. The voltage acting on the lamps at any instant is equal to the difference between the line voltage and machine voltage; if they are constantly equal and opposite this voltage is zero and so no current flows through the lamps and they remain dark. But if the machine frequency is not equal to line frequency the voltage on the lamps varies from a maximum (when the alternator and line act in series through the lamps) to zero, and so the lamps alternately glow and become dark.

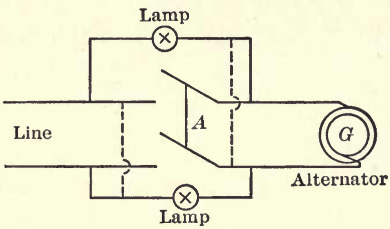


FIG. 40.

The reason for the intermittent glowing of the lamps becomes apparent when a vector diagram of the two E.M.F.'s acting in the local circuit is shown. Suppose that the two voltages are equal and that the line frequency is 60 and the generator frequency is 61; then the vector relations of the two E.M.F.'s may be shown by assuming the line frequency equal to zero and the generator frequency as one cycle per second. In Fig. 41 the line E.M.F. is shown at *OG*. The generator E.M.F. is shown in different phase positions with respect to *OG*, these different phase positions of generator and line voltage taking place as the generator "catches up" to the line voltage and then passes it, due to the higher frequency of the generator. The voltage acting on the synchronizing lamps is, of course, shown by the vector *OA*, *OA'*,

OA'', *OB*, etc. The locus of the extremities of these vectors will evidently be a circle with a diameter equal to the sum of the two E.M.F.'s. So that the E.M.F. acting on the lamps passes through all values between the sum and difference of the two E.M.F.'s, a number of times per second equal to the difference of the two frequencies.

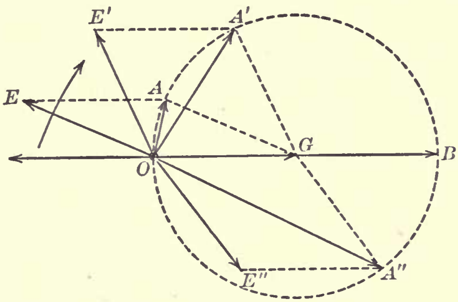


Fig. 41.

In the middle of a **dark** period the synchronizing switch *A* may be closed. Before the switch is closed the voltages of the machine and line should be tested for equality and the frequencies should be so nearly alike that the period of the lamp is approximately five seconds. If the lamps are connected as shown by the dotted lines in Fig. 40, the proper time for closing the switch *A* is at the middle of a **bright** period.

In general the alternators will not be of so low a voltage that the lamps may be directly connected between the bus and machine as shown. Where the voltage is higher than 220 volts a small step-down transformer is used for supplying current to the lamp, the primary of the transformer being connected where the lamp is shown in Fig. 40. Instead of using two transformers and two lamps, one lamp may be used and a small transformer fitted with two high-voltage windings and one low-voltage winding on a middle leg of the iron core. The connections for this method of using a synchronizing lamp are shown in Fig. 42, the synchronizing switch being shown at *A* and the transformer and synchronizing lamp at *B*. The transformer is so connected to the line and generator that when the two E.M.F.'s are in opposition practically no magnetic flux passes through the center leg of the core; the magneto-motive forces of the coils on the outside legs being in such relative directions that nearly all of the flux passes around the core through the outside legs and so induces no E.M.F. in the lamp circuit. If, however, the two E.M.F.'s are in phase (i.e. 180° from the proper phase for synchronizing) then the direction of the flux is as shown by the

arrows in Fig. 42; the center leg serves as part of the magnetic path for both outside coils. The lamp will be bright under such conditions, as there is a maximum voltage generated in the coil of the center leg when maximum flux passes through it.

In treating this question of parallel operation and some of the factors affecting it, the case considered is that which is met in commercial generating stations. Several alternators are connected in parallel and it is desired to determine the behaviour of one of the alternators when either its excitation or the torque of its prime mover is varied. As the machine under consideration is only one out of a number connected to the same line, it is to be first noticed that neither of the above variations can affect the terminal voltage of the alternator, this being determined by the rest of the machines in the station.

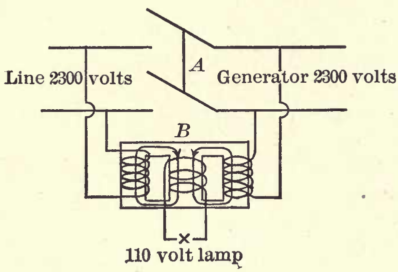


FIG. 42.

With this condition fixed, a vector diagram properly constructed serves very well to study the effect of variation in either the excitation or the torque of the alternator. In Fig. 43, OA represents the voltage of the station, to be designated by e. A lagging load is assumed and shown by OI. The generated voltage of the machine under consideration is equal to E and the armature-impedance drop equal to IZ. By using O and A as centers with E and IZ as radii, arcs are drawn, intersecting at D. Then DC is constructed making the angle α with DA, where $\cos \alpha = \dfrac{R_a}{\omega L_a}$ and R_a = armature resistance, ωL_a = armature reactance (including demagnetization). The line AB is drawn parallel to OI. Then this diagram represents the condition when the alternator is excited to voltage E, furnishing current I and generating total power P.

$$P = EI \cos \phi' = EI \sin (\delta + \alpha)$$

$$= EI (\sin \delta \cos \alpha + \cos \delta \sin \alpha),$$

$$\cos \delta = \frac{DF}{DA} = \frac{E - e \cos \beta}{I \sqrt{R_a^2 + (\omega L_a)^2}},$$

$$\sin \delta = \frac{AF}{AD} = \frac{e \sin \beta}{I \sqrt{R_a^2 + \omega L_a^2}},$$

so that $P = \dfrac{E}{\sqrt{R_a^2 + (\omega L_a)^2}} (e \sin \beta \cos \alpha + E \sin \alpha - e \cos \beta \sin \alpha).$

It is evident that $\dfrac{E}{\sqrt{R_a^2 + (\omega L_a)^2}} = I_0$, short-circuit current for excitation, E.

Therefore $P = I_0 e (\sin \beta \cos \alpha - \cos \beta \sin \alpha) + EI_0 \sin \alpha$

$$= I_0 e \sin (\beta - \alpha) + EI_0 \sin \alpha. \qquad (1)$$

This equation serves to explain the fact that when an alternator is first connected to the line it takes no load. Before synchronizing, the operator adjusts the field excitation of the incoming machine so that its voltage just equals the line voltage; i.e., in Fig. 43, $E = e$. Also the synchronizing switch is not closed until the phases of the line E.M.F. and machine E.M.F. are opposite; that is in Fig. 43, $\beta = 0°$. As the torque of the prime mover has been adjusted, before synchronizing, to just supply the losses in the incoming machine, after being synchronized there is no tendency on the part of the prime mover to change β. But when $\beta = 0°$ and $E = e$, equation (1) shows that the electrical power generated is zero.

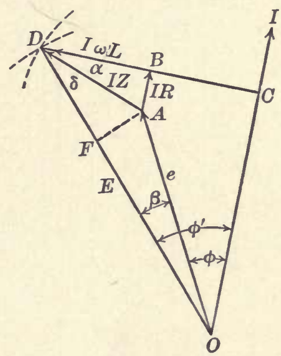


FIG. 43.

The angle β is the phase difference of the alternator E.M.F. and line E.M.F. If, after the incoming machine is connected to the line, the torque of the prime mover is increased it tends to speed up the alternator and actually does so for a fraction of a second, i.e., the alternator pulls ahead of the line E.M.F. and so gives to β some value other than zero. As the power that the alternator generates, hence the necessary torque of the prime mover, increases with increase of β, this momentary acceleration of the alternator lasts only long enough to give β such a value that $(I_0 e \sin (\beta - \alpha) + EI_0 \sin \alpha)$ equals the power developed by the prime mover.

The variation of load on an alternator as β is varied is readily determined experimentally; the results of such a laboratory test

are plotted in Fig. 44, which gives the curves of phase displacement with variation of load for three different excitations of the alternator. It may be seen from these experimentally determined curves that the increase in power output of the alternator is, within experimental error, proportional to its angular position, i.e., the value of β in equation (1). It will also be noticed that when the generator is overexcited ($E>e$) the variation of β, for

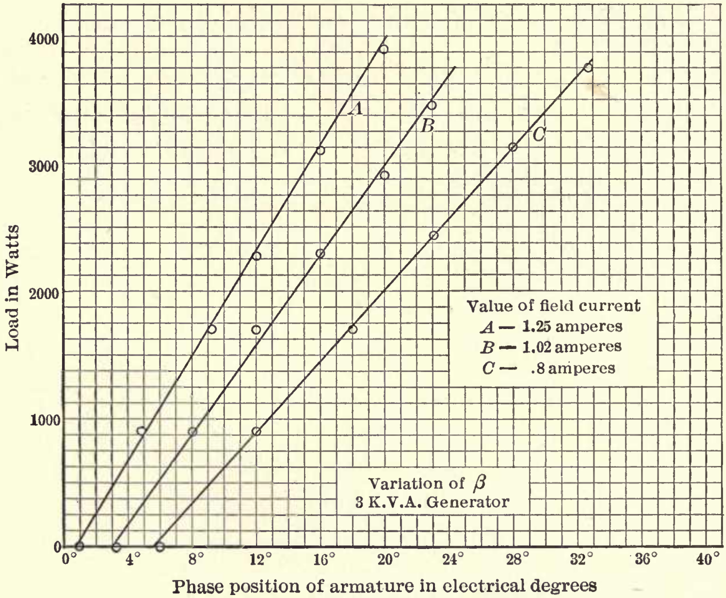


FIG. 44.

a given increase in load, is less than on normal excitation and for subexcitation ($E<e$), this variation in β is greater. This would, of course, be predicted by investigating equation (1). If E and I_0 are both increased, caused by increased excitation of the alternator being tested, a correspondingly less increase in β is required for a certain value of P.

If the prime mover does not exert a uniform torque (the case with all reciprocating engines) the value of β will be more or less variable. The frequency of its variation will be the same as the frequency of variation of the torque of the prime mover; the amplitude of the variations of β will depend not only upon the variations in the driving torque, but upon the electrical characteristics of the alternator. If β increases, so does the load of

the alternator; hence a great increase in driving torque may not produce much change in the phase position of the alternator. In the curves given above, it is seen that, for normal excitation, a phase displacement of 20° (electrical) was sufficient to change the alternator load from zero to full-rated value.

A periodic variation of β (termed **hunting**) is undesirable because under certain conditions it may become so excessive as to throw the alternator out of synchronism with the line; hunting is also likely to cause disturbances in the other synchronous machines on the line. It is evident that to keep the variation in β small for a given variation in the driving torque, the alternator should have a large variation in power output for a small variation in β. From equation (1) we have:

$$\frac{\partial P}{\partial \beta} = eI_0 \frac{\partial}{\partial \beta}(\sin(\beta - \alpha)) + EI_0 \frac{\partial}{\partial \beta} \sin \alpha$$

$$= eI_0 (\cos \beta \cos \alpha + \sin \beta \sin \alpha)$$

$$= eI_0 \cos(\beta - \alpha).$$

Now $$I_0 = \frac{E}{\sqrt{R_a^2 + (\omega L_a)^2}} = \frac{E}{R_a} \frac{R_a}{\sqrt{R_a^2 + \omega L_a^2}} = \frac{E}{R_a} \sin \alpha.$$

If $\beta = 0$

$$\frac{\partial P}{\partial \beta} = \frac{eE}{R_a} \cos \alpha \sin \alpha = \frac{Ee \sin 2\alpha}{2 R_a}.$$

This expression is termed the synchronizing power of an alternator. It is a measure of the effort of the alternator to maintain a constant phase relation with respect to the E.M.F. of the line. It will evidently be a maximum, for a given value of R_a, when $\alpha = 45°$, i.e., $R_a = \omega L_a$.

In a good commercial machine ωL_a is much larger than R_a. For a given amount of copper in the armature, to make $R_a = \omega L_a$, would mean low efficiency and poor regulation. To give a large synchronizing power to a machine, therefore, ωL_a should be decreased as much as possible but never to such a low value that $\alpha < 45°$.*

* This idea of synchronizing force is not as simple as indicated above when there is much fly wheel effect to be considered. If the prime mover has a variable torque and the rotating member has a large fly wheel effect, it may be advisable to make the synchronizing force very low; in other words ωL_a should be very much greater than R_a.

We next investigate the effect upon the machine of changing its excitation and the first thing to consider is the load. Suppose that $E = e$ and that $\beta = 0$, which condition holds just after the machine is synchronized. The machine E.M.F. and line E.M.F. may be shown as in Fig. 45. Now suppose that E is decreased so that the resultant of E and e is shown by OC. The current produced by OC will flow through the armature of the test machine and then through all the other armatures in parallel. As e is taken as line voltage it is evident that OC must all be used

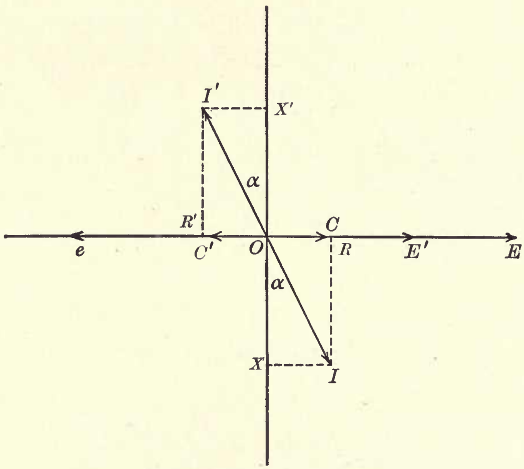


FIG. 45.

up as IZ drop in the armature of the test machine. The current I will lag nearly 90° behind OC because of the comparatively high inductance of a good armature noted above, we have $\tan\alpha = \frac{IR_a}{I\omega L_a}$, and the only power which this current OI represents is $EI \sin\alpha$. But this is the power lost in heating the armature due to the resistance loss. It is, therefore, evident that increase of excitation will not make the incoming machine take its share of the load.

The current component OX, it will be noticed, is wattless and represents no power output. It lags 90° with respect to OE and hence will act as a demagnetizing current for the overexcited alternator and will tend to magnetize the other machines on the line, but we have supposed the other machines to be of such capacity that this magnetizing action is negligible.

In case the excitation of the test machine is decreased the resultant voltage will have the phase OC'. The circulating current is OI', having a watt component OR' (supplied to the test machine by the line) and a wattless component OX'. The phase of OX' is opposite to that of OX; it will tend to magnetize the test machine.

A complete vector analysis of this question of variable excitation is possible by using the relations expressed in equation (1).

$$P = I_0 e \sin(\beta - \alpha) + EI_0 \sin \alpha.$$

Multiply each side of this equation by $\dfrac{E}{I_0 \sin \alpha}$ and add to each side $\left(\dfrac{e}{2 \sin \alpha}\right)^2$.

Also substitute for $\sin(\beta - \alpha)$ its equal $(-\cos(90° + \beta - \alpha))$. This gives,

$$E^2 + \left(\frac{e}{2 \sin \alpha}\right)^2 - \frac{Ee}{\sin \alpha} \cos(90° + \beta - \alpha) = \frac{PE}{I_0 \sin \alpha} + \left(\frac{e}{2 \sin \alpha}\right)^2. \qquad (2)$$

Considering the right-hand side of this equation:

We have just seen that P is independent of excitation; $\dfrac{E}{I_0} =$ armature impedance, which is supposed constant, and we have supposed e constant. Also $\sin \alpha$ is constant.

We may therefore write

$$E^2 - \frac{Ee}{\sin \alpha} \cos(90° + \beta - \alpha) + \left(\frac{e}{2 \sin \alpha}\right)^2 = K^2 \qquad (3)$$

where

$$K^2 = \frac{PE}{I_0 \sin \alpha} + \left(\frac{e}{2 \sin \alpha}\right)^2 = \frac{P(R_a^2 + (\omega L_a)^2)}{R_a} + \frac{e^2 (R_a^2 + (\omega L_a)^2)}{4 R_a^2}$$

$$= \left(\frac{4 PR_a + e^2}{4 R_a^2}\right)(R_a^2 + \omega L_a^2); \text{ (independent of } E).$$

Now an equation in the form of (3) represents a triangle having sides E, $\dfrac{e}{2 \sin \alpha}$, and K, and the angle between E and $\dfrac{e}{2 \sin \alpha}$ is $(90° + \beta - \alpha)$. To construct the triangle erect OA (Fig. 46) perpendicular to the base line and equal in length to e. Construct OB at an angle α to the base and intersecting at B, the line constructed perpendicular to OA at its middle point. With a radius equal to K and center at B construct an arc.

Through O draw a line making the angle β with OA and intersecting the arc at C. Then OBC is the triangle expressed by the equation (3) of which the side OC is the generated E.M.F. of the alternator. As OA is the line voltage e, the line AC represents the impedance drop in the armature. For various excitations the point C will lie on the circular arc as locus, and the current will be proportional to the length of AC and constantly at the angle $(90 - \alpha)$ to AC.

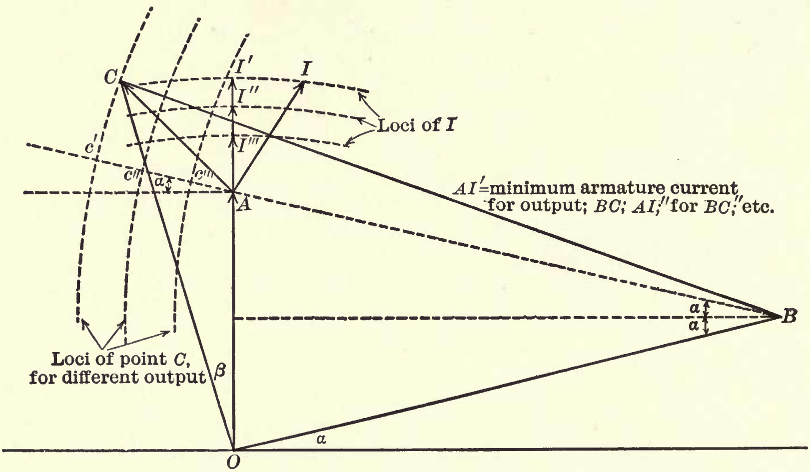


FIG. 46.

The current is shown at AI; the locus of I is easily constructed and gives, in polar coördinates, the variation of the armature current of an alternator with variations of generated voltage OC.

For different power output the length of the line BC has a different value, but the rest of the construction is identical. The loci of the current are given in Fig. 46 for three different loads. They show that, for a given watt output, there is a certain excitation which gives a minimum current output and that this proper excitation increases slightly with increase of load.

The previous discussion holds for cases in which the test alternator does not affect the line voltage or the magnetic fields of the other machines. Two alternators of approximately the same size, operating in parallel, will not give results exactly as outlined above because one machine influences the other very

much, and the line voltage varies with the excitation of either machine.

It is a very interesting and instructive test to actually measure the angle β (equation 1) for different loads and excitations of the alternator, in order to see whether or not the results reached theoretically may be experimentally verified. A very simple method is here given for measuring β; it is based on the assumption that whatever manipulations are carried out on the alternator being tested the field distribution of one of the alternators supplying power is not distorted from its normal position.

It will be remembered that our investigation of the effect of armature reaction showed a cross-magnetizing effect, the magnitude of which depended upon the current being carried by the armature conductors and upon the power factor of the load. We would, therefore, expect a shifting of the field of the alternators supplying power as the conditions of load and field excitation of the test machine are changed. But if the machine being tested forms only a small fraction of the total capacity of machines connected to the line, this field shifting will be negligible. The machine used for obtaining the curves given in Fig. 44 had a capacity of 3 k.v.a. while the alternator supplying it with power was of 30 k.v.a. capacity and in addition had a very stiff field.

The scheme for measuring β may be understood by reference to Fig. 47. Two discs of some insulating material are used, one on the shaft of the machine supplying power and the other on the shaft of the machine being tested. These discs are fitted with conducting strips running from the center conducting drum to the outside of the disc; they are just the same as that described in Experiment 1, used in the plotting of curves by the method of instantaneous contacts. The brush bearing on the periphery of disc *A* (Fig. 47) is fixed while that on the shaft of the test alternator is movable over a graduated arc. It is quite evident that the lamp will burn only when the brushes in *A* and *B* both make contact with the metallic strips at the same instant. A voltmeter may be used instead of the lamp, if desired. The two discs are, of course, rotating synchronously (if the two machines have the same number of poles), so it is evident that by shifting the brush of machine *B* a position may be found such that the circuit, containing the lamp, is closed once each revolution and the lamp will burn. A condenser shunted across the

lamp terminals will increase the brilliancy with which the lamp burns. Why? It may be that the two machines do not have the same number of poles. The discs work most effectively when there is one strip for each pair of poles of the machine on which it is mounted. If the motor is a six-pole machine, e.g., there

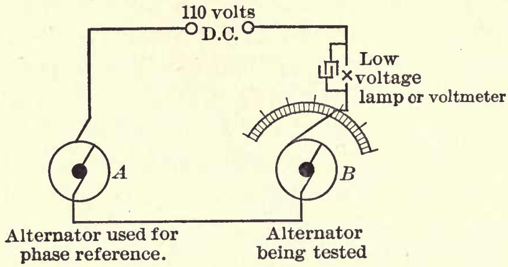


FIG. 47.

should be on disc B three strips, spaced 120° (mechanical) from each other.

Suppose the movable brush of machine B is set, with no load on the alternator, so that the lamp burns. If now the torque of B's prime mover is increased it will be found that the lamp no

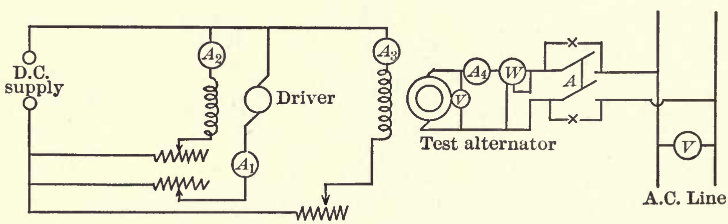


FIG. 48.

longer burns; the movable brush of B must be shifted forward a certain amount before the lamp is again bright. The amount that the brush must be shifted serves to measure the change in angle β; if the brush position for $\beta = 0°$ is desired it may be found by finding the brush position to make the lamp burn when there is no exchange of power between machine B and A and when B's field is adjusted to give an E.M.F. just equal to the line E.M.F.

Set up apparatus about as shown in Fig. 48; using lamps or synchronoscope, bring the alternator into synchronism with the line and note the current in both field and armature of driving motor. Close switch A and note readings of all instruments. Keeping the motor field at the value just read, vary the alternator excitation both above and below normal, using for the limit of range that field current which gives 25 per cent overload current in A_4. Get readings of all instruments at about five points above and five points below normal alternator field; read also phase position of armature for each value of field current.

With normal field (i.e. the field current which makes the power factor as nearly equal to one as possible; this current will be different for the different loads) on alternator increase the driving torque (by decreasing A_2) in steps until load on alternator is 25 per cent overload. Get about six readings of all instruments, and phase position of armature, in the range given.

With half load and with full load on the alternator, take a series of readings with different alternator excitations as given above.

Make a set of tests using instead of the A.C. line another alternator of about the same capacity as the one being tested. With normal field current on both machines, put on the line sufficient load to bring the load on both machines up to rating. Adjust the prime movers to make the alternators divide the load in the ratio of their capacities. Adjust the field excitations to make the load current equal to the sum of the machine currents (if possible). Then leaving all adjustments fixed, decrease the load to zero in about five steps, reading output of each alternator and load current and line voltage.

It will be noted that the division of load between two alternators under these conditions depends not upon the shape of the external characteristics of the alternators, but upon the form of the speed-load curves of their prime movers.

Calculate power factor of the alternator for all readings.

Plot curves as follows (for tests with line voltage constant):

On one curve sheet, with alternator field current as abscissæ, plot alternator armature current, watts output, and power factor for the three runs in which the torque of the prime mover was maintained constant and the test alternator was connected to the constant voltage line.

On a second sheet plot the relations of torque of prime mover,* watts output and alternator field current to phase position of alternator armature, using phase position as abscissæ.

On a third curve sheet plot the results obtained from the test carried out with alternators of equal capacities. Using load current as abscissæ give curves of watts output and power factor of each machine, line voltage and circulating current. Circulating current may be obtained by use of the formula

$$\text{Circulating current} = \sqrt{I^2 - \left(\frac{W}{E}\right)^2} \quad \text{where}$$

W, E and I are the readings of the wattmeter, voltmeter and ammeter in the armature circuit. Explain this formula.

Explain all results using vector diagrams where possible.

Note. — All of the previous tests upon the alternator have supposed a single-phase machine. If desired, some of these tests may be deferred and run on a polyphase alternator, after the preliminary tests on polyphase power have been performed.

* For the "torque of prime mover" may be used the product obtained by multiplying together the armature and field current of the driving motor. This product is nearly proportional to the true value of torque. Why?

EXPERIMENT XVII.

STUDY OF THE CURRENT AND E.M.F. RELATIONS IN A CONSTANT POTENTIAL TRANSFORMER ON NO LOAD AND ON FULL LOAD.

A CONSTANT potential transformer consists essentially of two coils of wire, insulated from one another, placed on a laminated iron core, in such a way that M, their coefficient of mutual induction, is as nearly as possible a constant maximum value.

If a sine wave of E.M.F. is impressed upon one coil of a transformer, the other coil being on open circuit, the equation for current in the excited coil may be obtained by solving the equation which expresses the relation between impressed force and reacting forces:

If R = ohmic resistance of coil excited.
F = the field set up.
$E \sin \omega t$ = impressed force.
x = current, the general equation will be of the form

$$E \sin \omega t = Rx + \frac{d}{dt}(F).$$

The expression Rx gives the reacting force due to the ohmic resistance of the conductor and $\frac{d}{dt}(F)$ expresses all of the reactions caused by the variations in the magnetic field. In this term will be included a dissipative reaction due to hysteresis and eddy currents in the iron, a dissipative reaction due to the secondary current (if there should be one) and a nondissipative reaction ordinarily termed the transformer reactance, which is caused by the field flux cutting the conductors of the primary coil.

The above equation is not in a form which is solvable, so we must express $\frac{d}{dt}(F)$ more in detail. When there is no secondary current the equation of reactions may be put

$$E \sin \omega t = Rx + L\frac{dx}{dt} + x\frac{dL}{dt} \tag{1}$$

where

x = no-load current.
R = ohmic resistance of primary coil.
L = coefficient of self-induction of the coil.

Now L is defined by the equation:

$$L = \frac{.4\,\pi S^2 A\mu}{l} \tag{2}$$

where S = number of turns in coil.
A = area of magnetic path.
l = mean length of magnetic path.
μ = permeability of iron.

Now $$\phi = \frac{.4\,\pi S x A\mu}{l}. \tag{3}$$

By investigation of this formula (3), in connection with a hysteresis loop as given in Fig. 52, it is seen that μ has no real meaning when the iron is going through cyclic changes in flux density. When the magnetizing force is zero the flux still has a considerable value, giving for the value of μ, ∞. Where the hysteresis loop crosses the axis of M.M.F. the flux has zero value, whereas the magnetizing force is not zero. Here the value of μ is zero.

A sort of average μ may be obtained experimentally by the use of formula (3). If the effective values of ϕ and x are known from test, and used in this formula, a value of μ is obtained which, although its meaning is not obvious, may be used for calculation in formulæ similar to (2). But such a value of μ may not be used in equations involving it as does equation (1). The term $\left(L\frac{dx}{dt} + x\frac{dL}{dt}\right)$ must be such a function, that, integrated over one cycle, it gives the area of the hysteresis loop. As neither L nor x are expressible as simple functions of the time it is evident that the solution of the equation is not possible without making some assumptions.

If L is assumed constant, equation (1) may be solved and gives

$$x = \frac{E_m}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t - \theta) \text{ in which } \tan\theta = \frac{\omega L}{R}.$$

In commercial transformers, R is very small compared with ωL, and θ would be nearly 90° if it were not for the core losses

which have already been mentioned. The hysteresis and eddy-current losses in the core must be supplied by the exciting current.

These losses result in shifting the phase of the current with respect to the impressed E.M.F., so that in practice the value of θ may be between 70° and 80°.

In equation (1), assuming that L is a constant, the term $L\frac{dx}{dt}$ is so large compared to R that it may be put, without much error, $E = L\frac{dx}{dt} = S\frac{d\phi}{dt}$, which gives the angle between the flux and impressed force as 90°.

The investigation of the reactions in the transformers from the standpoint of differential equations will not be carried further, but the author wishes to point out the fact that the ordinary treatment of the transformer using the differential equation of reactions is not complete. The circuit as ordinarily depicted does not provide for the iron loss in the transformer, and, moreover, it cannot be made to do so. If a complete mathematical analysis of the reactions is attempted, the transformer circuit must be imagined as made up of two primary circuits instead of one. This is shown in Fig. 49, in which the

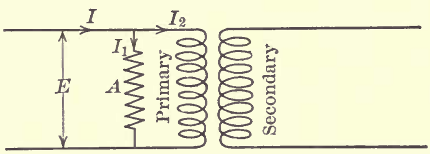


FIG. 49.

path A is of such resistance that the power absorbed in it is just equal to the iron losses in the transformer. Then the line current on the primary side is made up of two components, I_1, in phase with E and independent of load, and I_2, which itself will consist of two components; the one component will serve to magnetize the field and will be nearly 90° behind the impressed E.M.F. in phase (not exactly 90° because of the ohmic resistance of the primary coil); the other component will be in opposition with the secondary current and will be of sufficient magnitude to supply a M.M.F. just equal and opposite to that produced in the secondary coil by any current that may be circulating in that coil.

Even if the problem is attacked (by mathematical analysis) with the above represented equivalent circuit in place of the real circuit, an exact solution cannot be reached because of the variability of L (noted above) as the iron goes through the different parts of the hysteresis loop. Also in the actual circuit of the transformer, the iron losses vary to a slight extent as the load is increased, this effect is not accounted for in the circuit of Fig. 36.

The E.M.F. in the secondary coil is caused by the variation of the flux* and so must be in phase with the primary reacting force

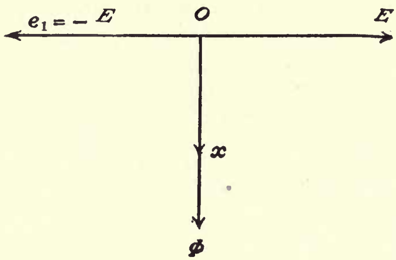


FIG. 50.

caused by the flux variation. But at no load this reaction is approximately 180° out of phase with the impressed force, so that the secondary E.M.F. is also 180° displaced from the impressed E.M.F. A vector diagram showing these relations is given in Fig. 50. Here the effect of losses in the iron core has been neglected and also the IR drop in the primary is neglected.

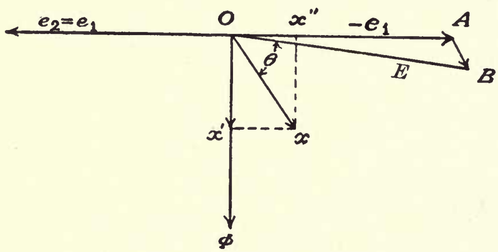


FIG. 51.

Actually the iron losses in a transformer are of such a magnitude that Fig. 50 does not represent the case with sufficient accuracy and the relations are really as shown in Fig. 51. The exciting current x lags behind the impressed force E by the angle θ. The component x' serves to magnetize the field and x'' supplies the no-load losses of the transformer. The field reaction in the primary is shown by $+e_1$, at right-angles to Φ; in phase with e_1

* See Appendix, Plate 7.

is shown e_2, the secondary E.M.F. AB, in phase with x, gives the "effective resistance" drop in the primary, so that OB must give the magnitude and phase of the impressed force E. It will be noticed, by comparison of Fig. 50 and Fig. 51, that the effect of the primary resistance is to slightly change the ratio of induced secondary E.M.F. to E.M.F. impressed upon the primary and to slightly change their phase relation. The first of these effects is very small when the transformer is not loaded, but the second is easily determined experimentally.

The vector analysis of the currents and E.M.F.'s when the transformer is loaded are left for the student to discuss.

It has been shown that if the magnetic reluctance of the transformer is constant the primary exciting current (primary current when secondary is open circuited) is of the same form as the impressed E.M.F. In the actual transformer the exciting current is far from being a sine wave. The reaction equation of the primary circuit when the resistance component is neglected (in discussing the current form this will be done) is

$$E_m \sin \omega t = S \frac{d\phi}{dt};$$

this shows that the flux must follow a cosine wave, $\phi = \phi_m \cos \omega t$.

Now by reference to the curve shown in Fig. 52, giving the cycle of magnetization of the transformer, it is evident that, if the current values for different values of ϕ on the cosine curve are taken from this magnetization curve, the current will have the distorted form shown in Fig. 53. Not only is the curve distorted but it reaches its zero value ahead of the flux by the angle X. The value of this angle X depends upon the width of the hysteresis loop of the transformer iron and the width of this loop is a measure of the energy loss in the iron. It will be seen that this angle X brings the exciting current more into phase with the impressed E.M.F., as it should do to represent energy consumption. The result of these differences between the ideal transformer and the

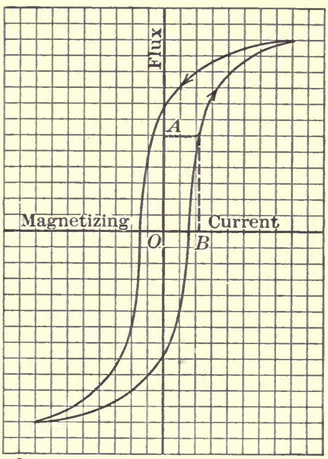


FIG. 52.

commercial one is that the exciting current instead of being a sine current lagging 90° behind the impressed E.M.F. has a peculiar peaked form and its zero value lags considerably less than 90° behind the zero of the E.M.F. wave.

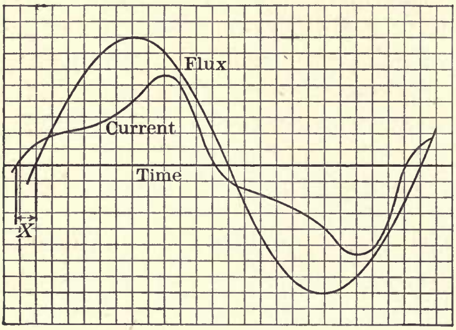


Fig. 53.

When the secondary circuit is loaded (suppose noninductively) then the current flowing in the secondary will be in phase with, and of the same form as, the secondary E.M.F. By the principle of conservation of energy there must be an equal and opposite current fed into the primary. Then the primary current will be the resultant of this load current and the exciting current. As the exciting current is only a small fraction of the full-load current of a transformer, when there is any appreciable load on the transformer, the primary current loses its distorted shape and comes very nearly into phase with the impressed E.M.F. On full load the primary current shows practically no distortion and is displaced only a few degrees from the impressed E.M.F.

With connections as shown in Fig. 54, obtain sufficient points on the three curves, primary and secondary E.M.F. and primary current, to get their proper forms. Then put full load on the secondary and get the four curves, E.M.F. and current for each circuit, both for inductive and noninductive loads.

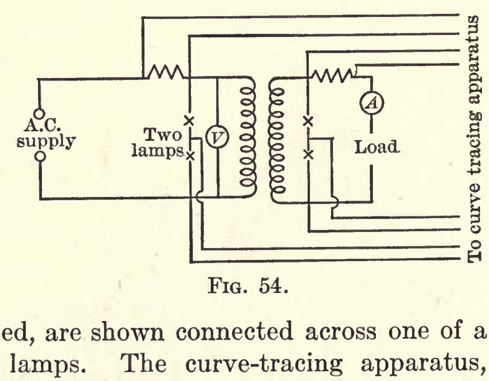


Fig. 54.

In Fig. 54 the leads, from which the voltage curves are to be obtained, are shown connected across one of a series of incandescent lamps. The curve-tracing apparatus, described in Experiment 1, permits the tracing of alternating E.M.F. waves of not greater than 110 volts maximum; the

transformer shown in Fig. 39 is supposed to give a 110–110-volt transformation so that the curves of primary and secondary E.M.F.'s could not be measured with the curve-tracing apparatus; their maximum values will be about 150 volts. So by connecting two lamps in series and taking the wave form of the potential difference across one of them the curve obtained is exactly similar to the E.M.F. wave desired, but of one-half the amplitude. Some such scheme as this is often necessary when using curve-tracing apparatus to obtain the forms of high E.M.F.'s or currents.

Caution. — Do not insert much resistance in series with the primary circuit, otherwise the exciting current will not have its characteristic form. Why? Could you measure the power factor of the exciting current from the curve sheet?

EXPERIMENT XVIII.

REGULATION AND EFFICIENCY OF A TRANSFORMER BY LOADING.

THE regulation of a transformer is defined as

$$\frac{\text{no-load voltage} - \text{full-load voltage}}{\text{full-load voltage}}$$

and its efficiency by $\frac{\text{output}}{\text{input}}$.

It is the object of this test to find these two characteristics of a transformer, and also the power factor, by loading the transformer and reading the proper quantities.

As the efficiency of a transformer is very high (generally over 95 per cent), a slight inaccuracy in reading or in calibration of instruments makes a very appreciable error in efficiency. If there is an error of 3 per cent in one of the meters the output may be read larger than the input. For this reason the method of losses, to be given in the next experiment, is much more accurate and to be preferred.

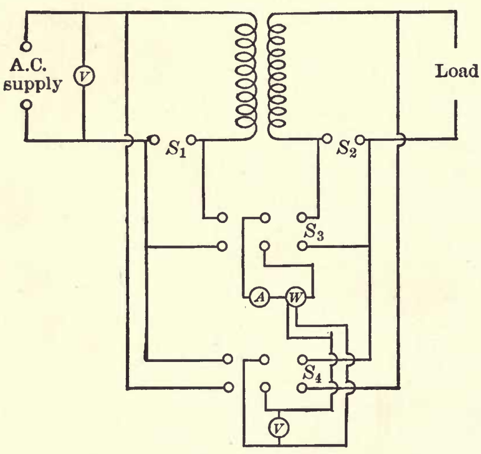


FIG. 55.

Wire up the transformer to be tested as indicated in Fig. 55. When the same meters are used in both circuits any inaccuracy in calibration will be eliminated in calculating efficiency. By reading the wattmeter, ammeter and voltmeter in the secondary circuit with noninductive load, the calibration curve of the wattmeter may be obtained by assuming the ammeter and voltmeter correct, and, of course, $\cos \phi = 1$. This procedure is justifiable

because in getting efficiency the same wattmeter is used in both circuits and in calculating the power factor absolute accuracy of the wattmeter is not necessary but it must be accurate **with respect to the product, volt-amperes** as obtained from the ammeter and voltmeter. The voltmeter V_1 is used merely for maintaining the impressed voltage constant and is left permanently on the primary circuit. No readings should be taken until the impressed voltage is exactly at that value which gives rated secondary voltage at full load, noninductive.

Adjust the impressed voltage so as to give rated secondary voltage when full-load, noninductive, current is flowing, and keep this value of impressed voltage throughout the test. See that the frequency is kept at that value for which the transformer was designed. Take readings of input and output for values of output as follows: 0, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 and $1\frac{1}{4}$ rated value.

Take similar readings for an inductive secondary load of $\cos \phi = 0.8$, keeping impressed E.M.F. the same as it was for $\cos \phi = 1$.

Calculate efficiency and power factor of primary circuit, for the different loads measured. Plot, on one sheet, efficiency, power factor and secondary volts for the noninductive load, using, for abscissæ, secondary load in amperes. On a second sheet plot similar curves for the inductive load. Calculate the regulation for both the inductive as well as the noninductive load.

EXPERIMENT XIX.

EFFICIENCY, REGULATION AND POWER FACTOR OF A TRANSFORMER BY THE LOSS METHOD.

THE two losses in a transformer are the core losses and the I^2R losses in the windings. The core loss is practically independent of load while the copper loss increases with the second power of the load.

As the transformer is loaded, leakage of the flux between the two coils occurs. This leakage flux is proportional to the load and it so produces eddy currents, etc., that it causes an extra core loss, termed the "load loss." In the customary method of performing this test the load losses are measured with the copper loss and are not separated therefrom.

The core loss consists really of two parts which may be separated from one another, hysteresis and eddy-current losses. The hysteresis loss depends only upon the quality of the iron, the maximum flux density and frequency, and would be the same if the core was made laminated or solid, i.e., this loss is not affected by laminating the iron except that thin plates of iron may be better annealed than solid pieces. To keep this loss small, special quality of magnetic steel is employed and the maximum flux density used is comparatively low.

The effect of the wave form of the E.M.F. impressed upon the core loss of a transformer has been investigated only recently. It is found that if two E.M.F.'s of different shapes but the **same effective values** are used in testing the core loss of a transformer, the losses will be different in the two cases. It is stated by one writer that the core loss of a transformer may vary 20 per cent under extreme conditions of wave distortion; e.g., tested with 110 volts (effective) on some commercial circuits the core loss of a certain transformer might be recorded as 100 watts, while, with the **same effective voltage** from a different circuit of wave form differing widely from the first, the loss might be as much as 126 watts.

A peaked E.M.F. wave will produce less hysteresis than a flat-

topped wave of the same effective value. This is really due to the fact that the hysteresis loss does not depend upon the average value of the flux density reached throughout a cycle, but upon the maximum value, and the flat-topped E.M.F. wave requires a higher maximum value of flux than does a peaked wave of the same effective value. The core loss of a transformer should be guaranteed only in connection with the wave shape upon which it is tested. The reason for the greater maximum value of the flux when the E.M.F. wave is flat-topped may be seen by examination of the reaction equation of the primary of the transformer. Neglecting the resistance reaction (which may be done without much error) it is seen to be

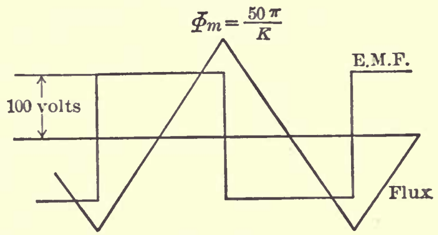


FIG. 56.

$$e = K\frac{d\phi}{dt}.$$

Now we will consider two E.M.F.'s waves of the same effective value, namely, 100 volts, as shown in Figs. 56 and 57. One is a rectangular and the other a triangular wave. The rectangular wave has a maximum value of 100 volts and the triangular one a maximum value of 172 volts.

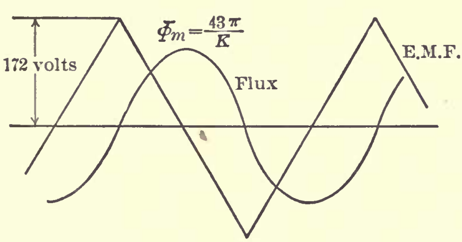


FIG. 57.

Using the equation given above we have

$$\phi_m = \frac{1}{K}\int_0^{\frac{\pi}{2}} e\,dt.$$

For the rectangular wave this gives $\phi_m = \dfrac{100\,\pi}{2\,K} = \dfrac{50\,\pi}{K}.$

For the triangular wave $\phi_m = \dfrac{1}{K}\displaystyle\int_0^{\frac{\pi}{2}}\left(\frac{2\times 172}{\pi}\right)t\,dt = \frac{344\,\pi^2}{\pi K\,8} = \frac{43\,\pi}{K}.$

The difference in ϕ_m in these extreme cases is seen to be about 14 per cent, and, of course, the hysteresis loss would be subject to

a much greater difference than this as it varies with the 1.6th power of the maximum flux density.

The eddy-current loss is kept down by using laminated iron for the core. The finer the laminations the less will be the eddy-current loss, as will become apparent by considering the effect of laminating. In Fig. 58 is shown a cross section of a core with laminations much thicker than is actually the case. The direction of the flux through the iron is taken as being perpendicular to the figure, so that the induced E.M.F. in the iron, due to flux changes, will be in the plane of the figure. The path of an eddy current is shown by the dotted line in lamination.

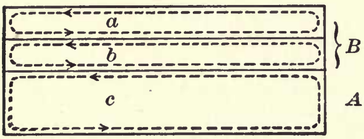


FIG. 58.

The magnitude of this current will be

$$I = \frac{E}{\sqrt{R^2 + X^2}},$$

where E = voltage induced in eddy path.
R = resistance of eddy path.
X = reactance of eddy path.

The loss will be $$I^2R = \frac{E^2R}{\sqrt{R^2 + X^2}}. \qquad (1)$$

Now only those paths in the laminations near the outside of the core can have any appreciable reactance. In all the laminations inside the magnetic field of the core the current in the eddy path will be in phase with the time variation of the flux, i.e., the reactance of the paths is zero.

We may, therefore, put

$$\text{Loss in lamination } A = \frac{E^2}{R}.$$

Now in laminations B (the two together of the same cross section as A) we shall have the relations $E_1 = \frac{E}{2}.$

$$R_1 = R \text{ approximately (apparent from diagram).}$$

Loss in both B laminations $= 2\,I_1^2R_1$

$$= 2\frac{E_1^2}{R_1} = 2\frac{\left(\frac{E}{2}\right)^2}{R} = \frac{E^2}{2\,R}.$$

So by finely laminating the iron the eddy-current loss in a commercial transformer is kept very low, being only a small fraction of the total core loss. If the core were not laminated the eddy-current loss would be many times the hysteresis loss.

The best quality of iron for transformer cores is one having a narrow hysteresis loop and a high ohmic resistance. It must also have a high value for μ.

The core losses are given in the form of an equation as

$$\text{watts (per unit volume)} = K_1 B_m^{1.6} f + K_2 B_m^2 f^2.$$

Where B_m = maximum flux density.
f = frequency.
K_1 = constant depending upon area of hysteresis loop of iron.
K_2 = constant depending upon ohmic resistance of iron.

The copper losses (ohmic) can be calculated when the resistance of each coil and the ratio of transformation is known. Each (current)^2 may be multiplied by its respective resistance; or the equivalent resistance $R_1 + \alpha^2 R_2$ (where α is the transformation ratio) may be multiplied by I_1^2. This method, however, does not include the load loss and so the copper loss is obtained in another way.

If the secondary winding is short-circuited, only a very small E.M.F. need be impressed on the primary to force full-load current through both windings. If the input in watts is measured under these conditions it will be the full-load I^2R losses + a certain small core loss. This core loss is probably greater than the actual load loss but is generally used as the load loss.

To get the iron loss, have a suitable ammeter, wattmeter and voltmeter in the primary circuit and **the secondary circuit open**, then impress a low value of E.M.F. upon the primary and gradually increase it until rated secondary E.M.F. is reached. Adjust the frequency and voltage of supply to proper rated value of the transformer. Read current, voltage and power supplied to primary circuit and voltage on secondary circuit. This input includes a very small I^2R loss in primary coil due to the exciting current. As this exciting current is generally less than 5 per cent of rated capacity and the copper loss varies with the square of the current, the copper loss for the exciting

current is of negligible value, hence this input is taken as the **core loss** which is assumed to be independent of load.

The reason for impressing at first a low value of primary E.M.F. and gradually increasing it, instead of throwing the primary at once on a line of normal voltage, is the protection of the ammeter and wattmeter in the circuit. With the secondary circuit open, after the steady state has been reached in the primary circuit, only a small current will flow, namely, the exciting current. As this current is about 5 per cent of the full-load current, the ammeter and wattmeter employed in this test will have a current capacity of not more than 10 per cent of the transformer rating.

The current, which flows in the primary circuit for a short time after switching to a circuit of normal voltage, however, may reach a value much greater than full-load current.* This excessive current, although only acting for a few cycles, is quite likely to cause serious mechancial injury to the instruments. The reason for the rush of current is given by the shape of the magnetization curve of iron and the reaction equation of the primary circuit. Figure 59 gives the magnetization curve of the core iron and the normal hysteresis cycle for the same. The maximum flux density normally reached is A.

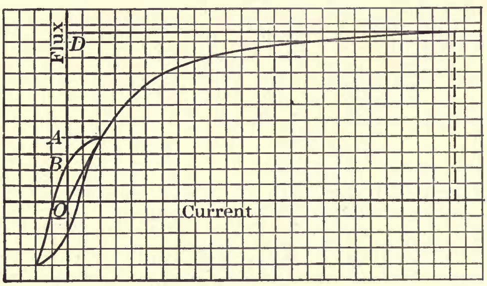


Fig. 59.

Now it may be that the power is switched off from the transformer just when the magnetism is at value A. The core will start to demagnetize along the upper side of the hysteresis curve, and the flux will continue to decrease until some point B is reached.

The reaction equation of the primary circuit is (when the resistance is neglected),

$$\text{Impressed E.M.F.} = K\frac{d\phi}{dt}.$$

Hence, so long as the impressed E.M.F. is positive, from A to B, Fig. 60, $\frac{d\phi}{dt}$ must be positive, i.e., ϕ must be increasing, and the total change in ϕ necessary to balance the impressed E.M.F. from

* See Appendix, Plate 8.

A to B (Fig. 60) = twice the maximum flux density in the core during normal operation. If then the core is left with a flux = OB (Fig. 59) and the transformer is switched to the supply circuit when the E.M.F. is zero and increasing (time A, Fig. 60) at time B the flux must have reached a value OD (Fig. 59) where $BD = 2\,OA$.

It is quite evident that this is so far beyond the saturation point in the iron that the inductance reaction becomes nearly zero. Hence the impressed E.M.F. is balanced only by the IR reaction, which, as the resistance of a transformer is very low, necessitates an excessive value of current. If the transformer is switched to the supply circuit at time B (Fig. 60) the current taken during the succeeding cycles is scarcely more than normal value. The magnitude of the switching current may be anywhere between those given by the two conditions cited, depending upon the flux remaining in the transformer core and the phase of the E.M.F. when switch is closed.

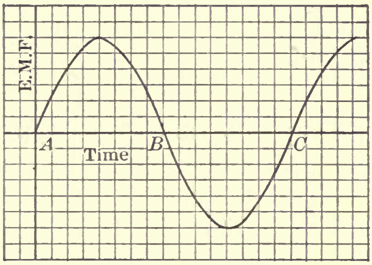


FIG. 60.

With the secondary short-circuited through an ammeter and suitable ammeter, voltmeter and wattmeter in the primary, impress a **very low voltage** on the primary. Gradually increase this until $\frac{1}{4}$-full-load current is flowing in secondary and read all instruments. Increase voltage and obtain similar readings for $\frac{1}{2}$-, $\frac{3}{4}$-rated and $1\frac{1}{4}$-rated secondary current. These inputs plotted with secondary current as abscissa give the copper loss and load loss curve.

These two losses may easily be separated; the ohmic resistances of the primary and secondary coils may be obtained by measurement on direct-current test and these resistances, multiplied by the square of the currents in the respective coils, give the actual copper losses of this transformer.

As a transformer is generally designed the copper losses in the two coils are equal, so that if the copper loss is calculated for one coil the total copper loss may be taken as double that amount. However, the resistances of the two coils must be measured. as sometimes the above relation does not hold.

After the ohmic resistances are known and the copper losses calculated and plotted in the form of a curve, the load losses are shown by the difference between this curve and that obtained in the A.C. copper-loss test. This is shown in Fig. 61. The load loss so obtained depends upon the amount of magnetic leakage between the two coils; if the coils are well laminated the load loss is small, but in no case can it be zero; unless the resistance of the coils is zero there must be some flux in the core while making the A.C. copper-loss test; this flux will give a load loss. It is to be noted that this method of obtaining the load loss is purely empirical and is probably far from being accurate. If the load loss was an important factor in obtaining transformer efficiency some better method for ascertaining its value could undoubtedly be formulated.

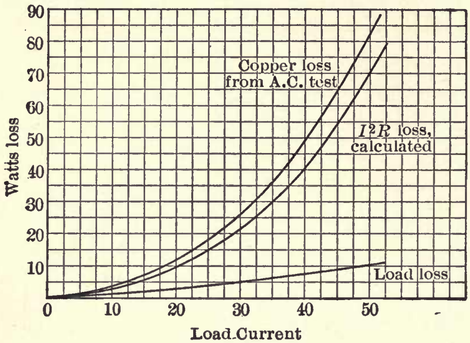


FIG. 61.

The different losses are to be plotted on one curve sheet, using load current as abscissæ.

The results obtained from the copper-loss test and iron-loss test may be used for predicting the behavior of the transformer on load.

The first characteristic to be predicted is the curve between secondary terminal voltage and load current. For the vector construction are needed two things: the open-circuit secondary voltage (with normal primary voltage impressed) as obtained from the iron-loss test; and the full-load impedance drop. These factors may be combined as in the simple diagram given in Fig. 62. OZ represents the full-load impedance drop, in terms of secondary E.M.F., plotted in proper phase with respect to the secondary current OI. With a radius of E_0, the open-circuit secondary voltage, an arc is drawn about O and then the terminal voltage at full load, noninductive, is given by the vector OE_t; at half-load by OE_t', etc. For inductive load of power factor = $\cos\theta$, the full load terminal voltage is obtained as shown at OE_t''. The terminal voltage for any load whatever is obtained by

subtracting (vectorially) from OE_0 the proper fractional part of the vector OZ.

The primary power factor is most readily predicted by the formula

$$\tan \phi = \frac{\text{total out-of-phase current}}{\text{total in-phase current}}.$$

If the load is assumed noninductive the only out-of-phase or wattless current is the magnetizing current while the watt current is equal to the energy component of the exciting current

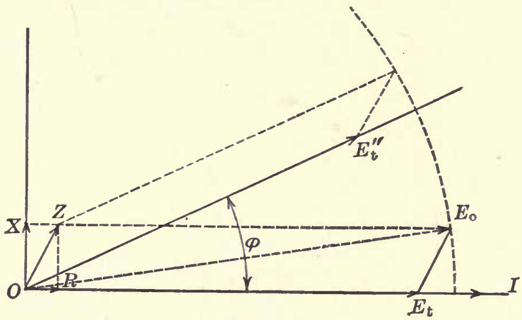


FIG. 62.

plus the secondary current (changed into its equivalent primary current by the known ratio of the transformer). If the load is assumed inductive, with power factor of $\cos \theta$, then the use of above formula for $\tan \phi$ necessitates the resolution of the load current into its two components, and each must be added to the like component of the primary exciting current. For no load the power factor is, of course, that obtained in the core-loss test.

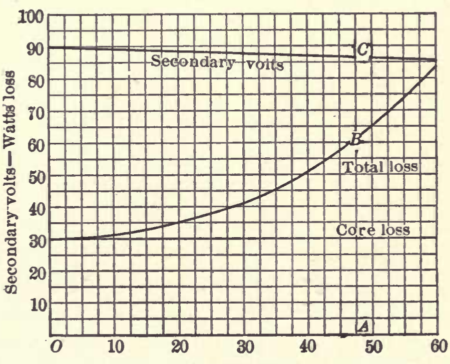


FIG. 63.

The next characteristic to be predicted is the efficiency, and the method employed is made clear by reference to Fig. 63, which shows the loss curves and the curve of secondary volts, obtained as just described.

At any assumed load OA, the loss is given by AB. The secondary output is equal to $OA \times AC$. The efficiency is, therefore, obtained as

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{OA \times AC}{(OA \times AC) + AB}.$$

Instead of using vector construction to obtain the external characteristic and power factor the different quantities may be combined analytically and, if very accurate results are desired, the analytical method is preferable.

EXPERIMENT XX.

VARIATION OF CORE LOSSES AND EXCITING CURRENT OF A TRANSFORMER WITH VARYING IMPRESSED E.M.F. AND FREQUENCY. SEPARATION OF IRON LOSSES INTO HYSTERESIS AND EDDY-CURRENT LOSS.

ALTHOUGH not of great value to the operating engineer the data obtained in this test is important to the designer, and also the test serves well to analyze the transformer losses.

The watts per unit volume used in the core losses are given by the equation:

$$\text{Watts} = K_1 B_m^{1.6} f + K_2 B_m^2 f^2$$

(Significance of symbols is given in Experiment 18.)

The value of the hysteresis exponent 1.6 holds only for flux densities up to 10,000 gausses; for higher densities the exponent becomes greater; for very low densities also this value of 1.6 does not hold. The value of K_2 will of course depend upon the resistance of the eddy-current path, i.e., upon the core temperature. It is probable that K_1 also depends upon the core temperature, but there seems to be no experimental data to show just how it varies.

Examination of above equation shows that if voltage is held constant, the core loss should decrease somewhat with increase of frequency. The eddy-current losses will remain constant (for a given impressed E.M.F., B varies inversely with the frequency) but the hysteresis will decrease because B is involved to the 1.6 power, while f is only to the first power. So that the total core loss will vary inversely with $f^{.5}$ (approximately).

For a given frequency the flux density B will vary directly with the impressed voltage (neglecting the resistance reaction in the fundamental equation for the primary circuit). The eddy-current loss will, therefore, vary directly as the second power of the voltage if the core remains at constant temperature, thus maintaining K_2 constant. The hysteresis loss will vary with the 1.6 power of the voltage. The total loss will, therefore, vary directly with the impressed E.M.F. to a power between 1.6 and 2, depending upon the relative magnitudes of the two losses.

The exciting current will be directly proportional to the impressed E.M.F. and will vary inversely with the frequency, so long as the reluctance of the magnetic circuit remains constant. If the circuit becomes more or less saturated then the current will change with the two variables to a power higher than the first. When the magnetic circuit becomes saturated a further increase in the E.M.F., or a decrease in the frequency, will make the current go to excessive values very rapidly.

For separation of the core losses into the two components, only two measurements of the watts are necessary, keeping the flux density constant. When the core is operated well below the saturation point, B varies directly with E. Hence, if the core losses are measured at two different frequencies, **keeping B constant** by keeping the ratio (E.M.F. $\div f$) constant, then K_1 and K_2 of the loss equation may be found. (If two different sources of E.M.F.'s are used, care must be taken that the form factors of their E.M.F. waves are the same.)

Eddy-current loss is the same for all frequencies, when impressed E.M.F. is constant, but varies with (E.M.F.)^2 when B is maintained constant, so we may put

Eddy-current loss $= aE^2$, where (a) is a constant.

Hysteresis loss varies with the frequency and with the (x) power of impressed voltage (x will ordinarily be 1.6, but the following demonstration is independent of its value).

Hysteresis $= bfB^x$ where (b) is a constant.

Then for the two values of frequency selected we have

$$W_1 = aE_1^2 + bf_1B^x.$$

$$W_2 = aE_2^2 + bf_2B^x.$$ B is the same in both equations when $\frac{E}{f}$ is held constant.

From these two equations the eddy current loss may be computed.

$$\text{Eddy-current loss (at } E_1) = \frac{W_1 - \frac{f_1}{f_2} W_2}{1 - \frac{f_2}{f_1}}.$$

This loss varies directly as (voltage)^2 and is independent of frequency, when impressed E.M.F. is constant, hence it may be readily calculated for any voltage other than E_1. Hysteresis loss

is then found by subtracting the eddy-current loss from total core loss.

Find the core loss and exciting current at normal voltage with normal frequency and values $\frac{3}{4}$, $1\frac{1}{4}$, $1\frac{1}{2}$, normal frequency. (If such variations in f are not obtainable use four other values in a smaller range.)

Find core loss and exciting current with normal frequency and values of voltage $\frac{1}{2}$-, $\frac{3}{4}$-rated and $1\frac{1}{4}$-rated value.

Obtain values of core loss at normal voltage and frequency at one-half normal voltage and frequency, from which calculate eddy-current loss at normal voltage. Calculate eddy-current loss at the other voltages used above and so calculate hysteresis losses for various conditions.

An exponential curve such as $Y = X^n$ becomes a straight line if plotted upon section paper having a logarithmic scale. The tangent of the line gives the value of the exponent.

Plot on logarithmic paper curves of hysteresis and total core loss for normal voltage and varying frequency. Plot similar curves for normal frequency and varying voltage. By measuring the tangent values prove that hysteresis varies with 1.6 power of voltage and first power of frequency. Upon curve sheet use watts for ordinates in both sets of curves.

Plot the curves of losses and of exciting current upon linear coördinate section paper.

If a transformer designed for 60 cycles is operated at 133 cycles, what could you predict as to the change in its characteristics, the supply voltage being the same in both cases? What if operated on 25-cycle current at same voltage?

EXPERIMENT XXI.

HEAT TEST OF A TRANSFORMER BY OPPOSITION METHOD; POLARITY TEST.

THE rating of an electrical machine is determined by the safe rise in temperature. A motor, in the construction of which is used ordinary insulation (i.e., perhaps cotton and shellac), might be rated at 10 H.P. The rating of 10 H.P. means that it is the maximum power the motor can deliver without overheating some of its parts. If now asbestos or other heat-proof insulation were used instead of cotton, the same size motor could be rated at perhaps 15 H.P. A certain percentage of the input of the motor is used up as heat in the motor itself; when this internal loss becomes large enough so that the heat generated can just be dissipated by the machine at a temperature which is safe for the insulation, etc., then the output of the motor under this condition should be its rated size.

The temperature-rise test is not taken for all machines sent out of a factory but only a few need to be tested, for if the machines have the same electric constants (resistance, etc.), and have the same radiating surface, they will behave alike in their temperature rise.

To determine the temperature rise of a machine caused by the losses which occur at full load it is not necessary to actually load a machine. If all of the full-load losses are being generated the temperature rise will be the same as though the machine were loaded. So different methods have been devised to furnish a machine with full-load losses without actually loading it. Generally it consists of testing at the same time two identical machines so connected together that one has generator action and the other motor action, the result being that each machine is operating under nearly full-load conditions yet the line is furnishing merely the losses.

In the case of two transformers this is accomplished by the "opposition" method, in which full-load current circulates through each transformer thus giving full-load copper loss, and

rated voltage is impressed on each of them so that normal core loss is generated. The method consists in connecting the low-voltage coils in parallel and impressing normal voltage; the secondaries are then connected in series (Fig. 64) with their E.M.F.'s opposing. Under this condition normal core loss occurs, but in the primaries there is only the exciting current and in the secondaries there is no current at all. If now the secondaries are opened and a source of variable E.M.F. is introduced into the circuit, full-load current may be caused to flow through both transformers (in both coils) by adjusting this E.M.F. to a value equal to the sum of the full-load impedance drops of both transformers. It will be seen that whereas the primary coils are in parallel (i.e., opposition) for the outside circuit supplying core-loss current, they are in series for the E.M.F. induced in them by the current which is caused to flow in them by the E.M.F. introduced into the secondary circuit. Hence full-load losses are occurring in the transformers and their temperature will rise as though they were supplying a load equal to their rating. The energy supplied is just equal to their full-load losses, hence this method can be used for obtaining efficiency also.

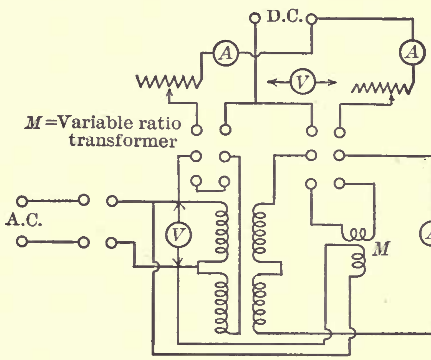


FIG. 64.

To know whether or not the secondaries are connected in opposition after the primaries have been connected in parallel, it is necessary to make a polarity test. Although the terminals of a transformer cannot be referred to as positive and negative, as they change polarity with the reversal of the alternating current, yet all terminals change their polarity together and so, **referred to one another,** the terminals may be said to have polarity. If the secondaries are so connected that at any instant their terminals which are connected together have the same polarity, then no current will flow through the closed circuit formed by their coils. If they are connected in the opposite manner the current will be the same as though each were short-circuited.

To test whether or not the secondary coils are in opposition connect together two of the terminals, one on each secondary, and measure the voltage between the remaining pair of terminals. This should be zero if in opposition, but may have a small value due to a slight difference in the ratios of the two transformers. The voltmeter used must have a range equal to twice the secondary voltage. With high-voltage secondaries where it is not convenient to use a voltmeter the circuit may be closed through a long, thin fuse, which, if the coils are in series instead of opposition, will blow before the short-circuit current can do any harm to the coils. The relative polarity of a primary and secondary may be tested by connecting as an auto transformer as in Fig. 65. If the voltage across AB is less than the voltage BD then A and C have opposite polarity and should be so marked.

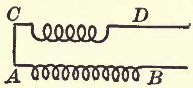


FIG 65.

The temperature of the coil, case and core are to be obtained by thermometers and that of the coils by resistance measurement. On the core and case fasten the bulb of the thermometer against the iron and cover with a piece of waste or putty to prevent radiation from the bulb and so that it may register the true temperature of the iron. Hang one thermometer in the oil, having the bulb near the top, as this will be the hottest part. If the thermometer has to be removed to be read, be sure to put the bulb back in the same place in the oil, otherwise very irregular readings will be obtained.

If the transformer has been standing some time (say 24 hours for a small one) since being used, the coils may be taken to be the same temperature as the oil when beginning the test. Measure their resistance before current has been passed through them long enough to heat them appreciably. Call this their resistance at the oil temperature. At any other resistance their temperature t may be calculated by the formula $R_t = R_0\,(1 + 0.0038t)$. Take readings every ten minutes until nearly uniform temperature is reached, when they need not be taken so frequently.

If the rise in temperature is small compared with the absolute temperature of the body the rate of radiation may be assumed as directly proportional to the difference in temperature between the body and the surrounding medium. Under such conditions the temperature rise will be a logarithmic curve and the final temperature rise may be predicted from the first part of the curve:

Assume T = temperature of the body.
H = specific heat $\times$ mass.
A = radiation per degree of temperature difference.
K = rate at which heat is supplied to body.
T_0 = temperature of surrounding medium.
t = time during which heat is supplied.

$$H\frac{dT}{dt} + A\,(T - T_0) = K.$$

$$\frac{dT}{dt} + \frac{A}{H}T = \frac{K}{H} + \frac{A}{H}T_0 = K'; \text{ or} \left(\frac{dT}{dt} + \frac{A}{H}\,T\right)e^{\frac{At}{H}} = K'e^{\frac{At}{H}}.$$

This is seen to integrate into the form,

$$e^{\frac{At}{H}}\,T = \frac{K'H}{A}e^{\frac{At}{H}} + K_0$$

in which K_0 is the integration constant.

$$T = \frac{K'H}{A} + K_0e^{-\frac{At}{H}}$$

$$= \frac{K}{A} + T_0 + K_0e^{-\frac{At}{H}}.$$

When $t = 0$, take $T = T_0$, then $K_0 = \frac{K}{A}$.

Therefore, $$T - T_0 = \frac{K}{A}\left(1 - e^{-\frac{At}{H}}\right).$$

Therefore, from two points in the first portion of the temperature-rise curve, the final temperature rise $\frac{K}{A}$ may be predicted.

Plot temperatures of the four parts measured for both transformers using time as the abscissa.

Caution. — After a resistance reading has been taken **be sure to disconnect the D.C. voltmeter before the D.C. circuit is opened.** The energy stored in the magnetic field of the transformer has to be discharged when the circuit is opened and if the low-reading voltmeter is across the coil it will furnish a path for this energy discharge and will probably be injured.

EXPERIMENT XXII.

STUDY OF THE CONSTANT-CURRENT TRANSFORMER AND DETERMINATION OF ITS CHARACTERISTICS.

FOR ordinary installations of power the supply circuit should be constant potential, and the constant-potential transformer is designed so that the ratio of primary to secondary E.M.F. is practically independent of load current. For some special purposes (notably, series arc lighting) a source of constant alternating current is desired, and to satisfy this requirement the constant-current transformer has been designed. On a series arc-light system when the number of lamps varies (e.g., some of them go out because of short carbons or other reasons) the current through the line must be maintained constant. But as the number of lamps in the circuit varies, the E.M.F. necessary to maintain constant current varies also, so that the requirement of the arc-light transformer is that it must maintain constant current while the resistance of the outside circuit is varied from zero to the value which gives rated capacity of the transformer. The equation of reactions in the two coils of a transformer are:

$$L\frac{dx}{dt} + Rx + M\frac{dy}{dt} = E\cos\omega t.$$

$$N\frac{dy}{dt} + Sy + M\frac{dx}{dt} = O.$$

L = coefficient of self-induction in primary.
R = effective resistance in primary.
x = current in primary.
y = current in secondary.
N = coefficient of self-induction in secondary.
S = effective resistance of secondary + outside circuit in series with secondary.
M = coefficient of mutual induction of the two coils.
$E\cos\omega t$ = E.M.F. impressed on primary.

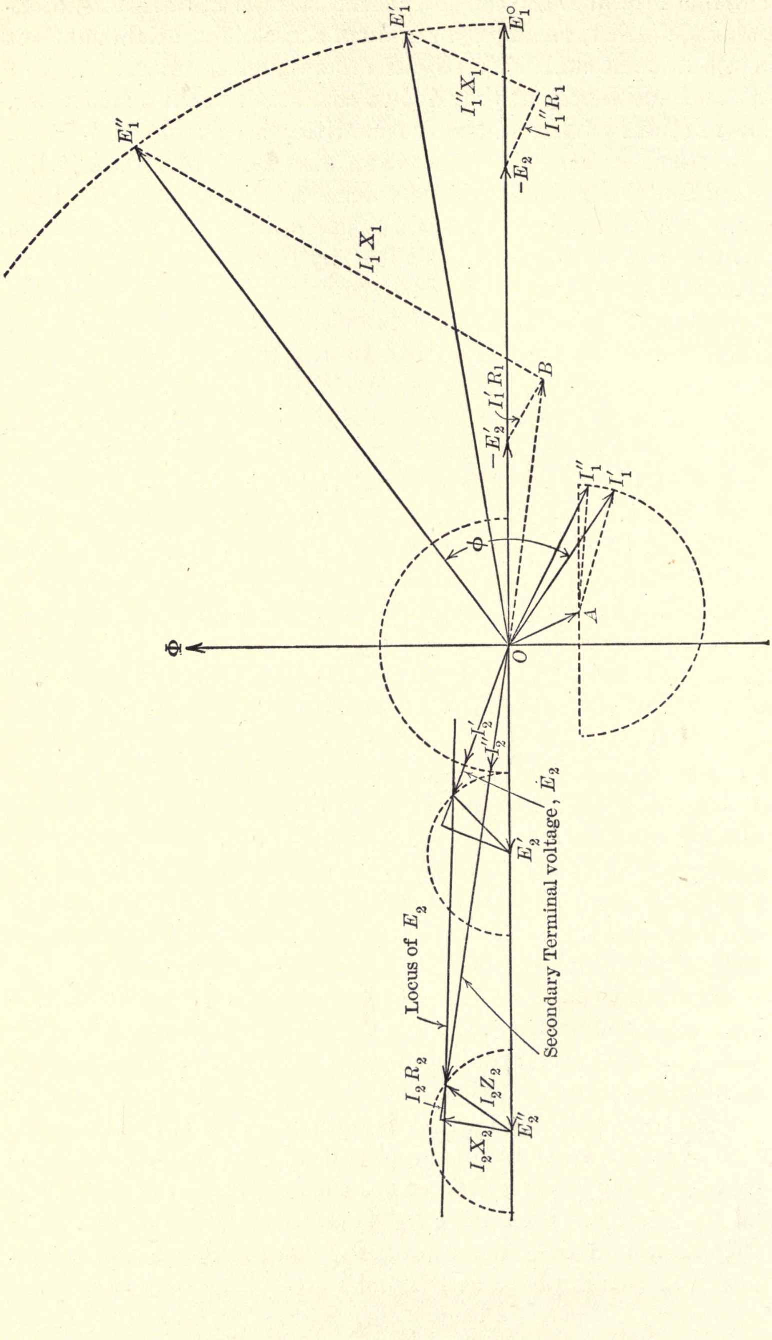
The constant-current transformer is made with the coils movable with respect to one another and it was found, in Experiment 5, that M varied with the relative positions of the coils.

When M and L are constant quantities (as in constant-potential transformer) the interpretation of these differential equations is not difficult and leads to the ordinary circle diagram. But when M and L vary as they do in a constant-current transformer, the relations of the different quantities involved can best be understood by analysis of the vector diagram as given in Fig. 66.

The flux threading both coils, when secondary is open-circuited, is represented in phase by Φ and the primary current, to produce this flux and supply hysteresis loss, by OA. Actually this flux, hence the current OA, will change somewhat for different loads on the transformer, but as OA is only a small part of the total primary current its changes will be neglected. The secondary impedance is assumed constant, which is another questionable assumption. However, a vector diagram constructed upon these two assumptions very nearly represents the actual case.

Suppose when M has its greatest value the transformer has a ratio of 1:1. The voltage impressed on the primary has the constant value given by the radius of the circle $E_1{}^0E_1'E_1''$. The current to be delivered by the secondary has the constant value given by the radius of the circle $I_2'I_2''$. Hence the primary current, being the vector sum of the exciting current and the secondary current reversed in phase, will be given by a vector from O to the circle I_1I_1'', which circle is constructed about A as a center and radius $= I_2$.

The generated secondary voltage (for a given value of M) is shown by the vector OE_2'. If, from this, the secondary impedance voltage I_2Z_2 is subtracted, the vector so obtained gives the secondary-terminal voltage OE_2. As we have assumed a non-inductive secondary load the I_2Z_2 drop must be so subtracted from OE_2' that its I_2R_2 component is in phase with the current I_2, the phase of which has not yet been obtained. The method of construction to satisfy these two conditions is to represent I_2Z_2 by its two components, then to change the position of I_2Z_2 until the I_2R_2 component if projected passes through O. Having now the secondary current (magnitude assumed and phase just obtained) the primary current is obtained at OI_1'. The three reacting forces in the primary circuit which must add vectorially to give the impressed force are the I_1R_1 drop, the I_1X_1 drop and the E.M.F. given by OE_2' reversed in phase. The I_1R_1 component and OE_2' are known in phase and magnitude and I_1X_1 is known in phase as it must be displaced 90° from OI_1'.



E_1''
E_1'
E_1°
$I_1''X_1$
$I_1''R_1$
$-E_2$
$I_1'X_1$
B
$I_1'R_1$
$-E_2'$
I_1''
I_1'
ϕ
A
Φ
O
I_2'
I_2''
Secondary Terminal voltage, E_2
E_2'
Locus of E_2
I_2R_2
I_2Z_2
E_2''
I_2X_2

FIG. 66.

Combining I_1R_1 and OE_2' gives OB. To OB is added a vector at right angles to the current OI'. This vector is continued until it intersects the circle $E_1'E_1''$ at E_1''. This gives the phase of the primary impressed E.M.F. and so the problem is solved. The power factor of the primary circuit is given by $\cos \phi$.

If it is assumed that OI_1 remains constant in magnitude (which it will do approximately) and the losses in the transformer are constant (copper losses are practically constant, iron losses increase slightly with increasing load) we may put $I_2E_2 = I_1E_1 \cos \phi = K$, which shows that as E_2 increases the power input to the transformer must correspondingly increase; as I_1 is constant this means that $\cos \phi$ increases directly with the load, which will be found nearly true for values of $\cos \phi$ less than 0.9.

The locus of the secondary terminal volts is given in Fig. 66, and it is seen that, throughout the working range of the transformer, this locus is nearly a straight line.

The diagram may also be satisfactorily constructed by supposing that all of the transformer inductance is in the primary circuit; the secondary coil having resistance only, its current and generated E.M.F. should be then considered in phase with each other. In many ways such a diagram gives a more logical idea of the transformer quantities than the one given here, which is similar to the usual transformer diagram.

Connect the primary to a source of constant potential, of voltage and frequency the same as transformer rating, and put in the circuit proper instruments for measuring the power input and relative phase of E and I. In the secondary circuit put an ammeter and voltmeter. (If an incandescent lamp load is used a wattmeter is not needed. If arcs are used for load, a wattmeter will be necessary.) A potential transformer will probably be necessary for the voltmeter on the secondary circuit.

After adjusting the counterweights so that rated current flows in secondary at full load, vary the load from zero to full load in about eight steps, reading all instruments and height of secondary coil.

Take two other runs, with the secondary underbalanced and overbalanced, to see how the transformer will regulate for other than rated current. Only secondary-current, position and secondary E.M.F. need be read for these two tests.

Construct a vector diagram of the E.M.F.'s in the two circuits for 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and full load on secondary.

Construct curves (with watts load as abscissæ) of efficiency, power factor, secondary E.M.F. and secondary current and position of secondary coil. On another sheet plot the curves of secondary current and position for the two conditions of incorrect counterbalancing.

How will the different losses in such a transformer vary with the load and why?

As the transformer efficiency is not very high and the power factor is very low at low values of secondary voltage, it is customary to use, on commercial circuits, transformers having several taps on the secondary coil, so that the transformer may be operated at high power factor on different circuits, having different numbers of lamps; the low voltage tap is used for a circuit having few lamps, etc.

EXPERIMENT XXIII.

PARALLEL OPERATION OF TWO CONSTANT-POTENTIAL TRANSFORMERS.

The conditions which affect the parallel operation of alternators, i.e., wave form, equality of voltage and phase, are also present to affect the operation of two transformers in parallel. In the case of the alternators, however, the attendant can regulate the conditions to reduce the cross current, etc., whereas, if transformers are connected in parallel to one feeder, the conditions cannot be readily changed. It is quite evident that if the transformers are to operate most efficiently there should be no cross current exchanged by their secondaries. If this is to be so, the instantaneous values of the induced secondary E.M.F.'s minus the respective impedance drops, due to load current, must always be equal, otherwise enough cross current will flow to bring about such a condition.

Considering first two transformers of the same capacity, it is evident that for such condition to be satisfied their respective impedances must be equal and they must have the same characteristic angle. This means that the respective resistances and reactances must be equal. The two transformers might have the same value, e.g., for full-load impedance drop, but if the reactance of one transformer is greater than that of the other (the corresponding resistance being less) then, although the terminal voltage of the two transformers will be of the same magnitude when full-load current is flowing, there will be a vector difference between the terminal E.M.F.'s which will cause a cross current to flow. Of two transformers connected in parallel the one having the poorer regulation will carry the smaller share of the load.

Now if it is desired to operate in parallel two transformers of different capacities, the full-load impedance drop (not impedance itself) must be of the same magnitude and phase in the two.

Connect two transformers of same design and capacity in parallel and vary the load in steps of about one-fourth of the

combined capacity of the transformers. Read the currents and watts output furnished by each and load (noninductive) current. Obtain the impedance and phase angle of the impedance drop for each transformer.

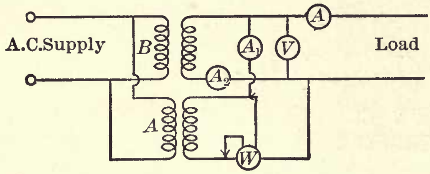


FIG. 67.

Perform the same tests for two transformers of different design and capacity.

Calculate the value of the cross currents for both cases, making the calculation at every one-fourth increase in load current. If connections are made as in Fig. 67, it is seen that the cross current may be obtained as follows:

W_1 = watts output of transformer A.
I_1 = total current output of transformer A, read on ammeter A.
E = voltage of load circuit, read on voltmeter V.

The in-phase current furnished by transformer $A = \frac{W_1}{E}$. The wattless or quadrature current of $A = \sqrt{I_1^2 - \left(\frac{W_1}{E}\right)^2}$.

Two 3-K.W. transformers of commercial design, supposed to be identical in all respects, gave results as shown in the curves of Fig. 68. A and B represent the currents supplied by each transformer to the load circuit and C is the cross current circulating between the two secondaries. The cross current was calculated by the formula given above and so represents the wattless current only. But the true cross current is not altogether wattless; at light loads, one transformer may supply considerable energy current to the other. This occurs in the above

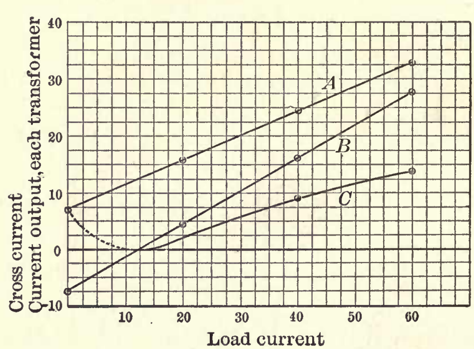


FIG. 68.

curves, when the load was 18 amperes; at lower values of load current there was not only a wattless current circulating between the two secondaries but a component of current in phase with the line voltage. The dotted portion of curve *C* represents a cross current made up of two components, a wattless current and an energy current. The above curves show the behavior of two transformers which had slightly different ratios (giving the no-load circulating current) and also different impedance. By adding external resistance to one secondary circuit and some inductance to the other these two transformers were made to divide the load equally at full load, but the equal division was not maintained when the load varied. The no-load circulating current could be brought to zero only by changing slightly the ratio of one of the transformers.

Explain the results of the test by vector diagram where possible.

If time permits, add either inductance or resistance to the transformer secondary circuits to see whether or not the load may be equally distributed and whether the equal distribution will be maintained as load is varied.

EXPERIMENT XXIV.

TWO-PHASE POWER, UNIFORMITY OF POWER, DIFFERENT METHODS OF CONNECTING, VECTOR ADDITION OF CURRENT AND E.M.F., POWER AND POWER FACTOR.

A POLYPHASE system is one in which are combined two or more single-phase E.M.F.'s differing in phase. In a symmetrical polyphase system the different E.M.F.'s are equal and equally displaced from one another in phase; in a balanced polyphase system the loads on the different phases are equal and the power delivered by such a system is uniform, whereas on a single-phase circuit the power supply is pulsating.

That the power delivered by a balanced polyphase system is uniform with respect to time may be easily shown. Consider a two-phase circuit, the voltage, current and power factor of each phase being the same.

Total power

$$= E_m \cos \omega t \, I_m \cos (\omega t - \phi) + E_m \cos \left(\omega t + \frac{\pi}{2}\right) I_m \cos \left(\omega t + \frac{\pi}{2} - \phi\right)$$

$$= E_m I_m (\cos \omega t \cos (\omega t - \phi) + \sin \omega t \sin (\omega t - \phi).$$

This gives by expansion:

$$\text{Total power} = E_m I_m \cos \phi,$$

which is evidently independent of time. In the same way a three-phase circuit may be shown to supply energy at a uniform rate.

The advantages of the polyphase systems over the single-phase systems are economy of material in machine construction (polyphase machinery, e.g., generators, motors and transformers, have less weight per K.W. capacity than single-phase machines) and economy in transmission-line construction. Also certain machinery, especially induction motors and rotary converters, practically require polyphase power for satisfactory operation. The single-phase induction motor, has, *per se*, no starting torque; the single-phase rotary has only a small percentage of the capacity of the same machine run polyphase and its operation is less satisfactory, etc.

Of all polyphase systems the two-phase is the simplest. It consists of two single-phase circuits interconnected or not, their respective E.M.F.'s being displaced 90° from each other. Such a system may be run four wire (in which case the two single-phase circuits may be entirely independent) or three wire, in which case a common wire is used for the two single-phase circuits. As this common wire carries more current than the other two it is made larger.

In case the phases are interconnected so as to give four E.M.F.'s equal and equally separated in phase it is styled the quarter-phase system. The connections, for two-phase three wire, are given in Fig. 69 (a); the two connections for the quarter-phase system, star and mesh, are given in Fig. 69 (b) and (c).

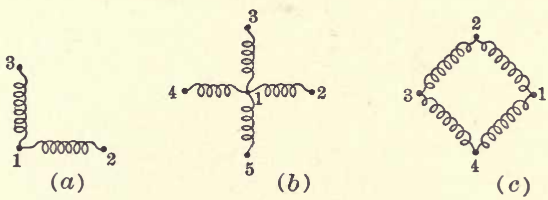


Fig. 69.

To study the E.M.F. and phase relation in the two-phase circuit make connections of phases as given in Fig. 70. Measure the voltage between 1–2 and 2–3 and 1–3. Then put a noninductive load on phase 1–3 and connect the pressure coils of the wattmeter to 3–2. From the volt-amperes on the wattmeter and watts indicated we may calculate the phase displacement between the voltage 1–3 and 3–2 (as a noninductive load is used, current in 1–3 is in phase with voltage 1–3).

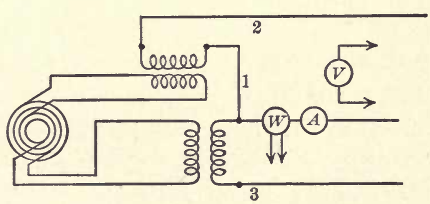


Fig. 70.

Put an equal noninductive load on each phase and measure the current in each line. (Use a dynamometer board and the same ammeter for the three readings so as to minimize calibration errors in instruments.) Measure the power supplied by each phase, using one wattmeter, connecting its current coil successively in each of the two outside lines and its potential

coil from each outside to the common wire as in Fig. 71 (a). Then measure the power with the same load by inserting the current coil of the wattmeter in the common wire and connecting its potential coil, one side to the common and the other to the two outer wires successively, as in Fig. 71 (b). Make the same tests with inductive load.

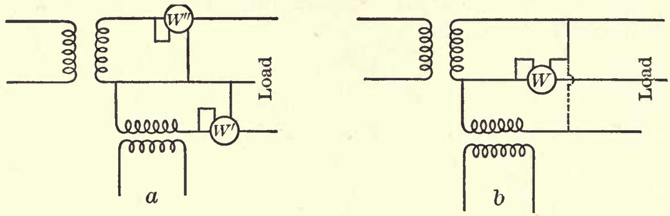


FIG. 71.

The power factor of the load may be found by the formula:

$$\text{Watts per phase} = (EI)\ (\text{per phase}) \times \cos\phi.$$

When the wattmeter is connected as in Fig. 71 (b), the potential coil being connected to upper line, the reading of the meter will give

$$W_1 = EI\sqrt{2}\cos(45° + \phi),$$

and with it connected to the lower line will read

$$W_2 = EI\sqrt{2}\cos(45° - \phi),$$

so that

$$W_1 + W_2 = EI\sqrt{2}\,(2\cos 45° \cos\phi) = 2\,EI\cos\phi.$$

This is evidently the true power of the system. It will be noticed that when the power factor of the system is not $= 1$, then the two wattmeter readings will not be alike; if the phase angle of the load is 45° one of the meters will read zero. If the lag or lead is more than 45° one of the meters will read negatively, under which condition its potential coil (or current coil) must be reversed, the reading taken and called negative, in which case the total power supplied $= W_1 - W_2$.

The power factor of the system may be obtained from wattmeter readings alone as shown by the following derivation.

$$W_1 + W_2 = EI\,2\sqrt{2}\,(\cos 45° \cos\phi),$$

$$W_1 - W_2 = EI\,2\sqrt{2}\,(\sin 45° \sin\phi),$$

so that

$$\tan\phi = \frac{W_1 - W_2}{W_1 + W_2}.$$

Calculate ϕ, for the inductive load, by this method and compare with value obtained by formula (1).

By means of the wattmeter prove that the two single-phase circuits do have their E.M.F.'s differing in phase by 90°.

Make the two connections for quarter-phase system and measure the different voltages. If one of the coils is connected wrong in the star connection (i.e., 180° out of phase), the different quarter-phase voltages will not be equal. In making the mesh connection, before the mesh is closed, measure the total E.M.F. in the mesh by connecting a voltmeter across the open " corner." This should be zero but will not be in case one of the coils is reversed. If the mesh is closed with one of the coils wrongly connected a large current will flow around the closed circuit. In each of the quarter-phase connections measure the diametrical voltage and each quarter-phase voltage.

Plot all E.M.F. and current measurements as vectors on section paper and explain the different results obtained.

EXPERIMENT XXV.

CURRENT AND E.M.F. RELATIONS IN A THREE-PHASE CIRCUIT; POWER AND POWER FACTOR; "EQUIVALENT" RESISTANCE AND CURRENT.

A THREE-PHASE system is one in which the voltages generated in the three-phase windings, from which the three-phase system is obtained, are 120° apart. That this angular displacement of E.M.F.'s is so follows from the placing of the coils on the armature. These three single-phase windings may be connected together in **star** (or Y) shown in Fig. 72 (a); or they may be connected in **delta** (or mesh) shown in Fig. 72 (b). When connected in delta the line voltage is quite evidently the voltage generated per

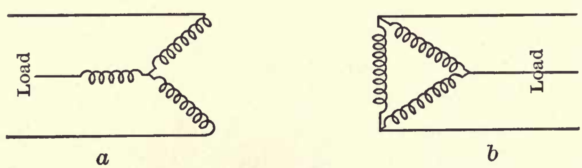


FIG. 72.

phase. In the Y connection this is not so and the relation of line voltage to phase voltage constitutes the first part of this test. The three-phase voltages are represented by equal vectors spaced 120°.

If equal noninductive loads are connected to each of the three phases it is evident that the three lines 1′, 2′ and 3′, in Fig. 73, will be carrying equal currents 120° out of phase with each other. But the sum of three equal vectors 120° apart is zero. Hence the three lines 1′, 2′ and 3′ may be joined together for their whole length and no current will flow in the resulting conductor. But if no current flows in it the conductor is useless and so is not used and we have the normal three-wire Y connection given in Fig. 72 (a). The current in line 1 will be in phase with the E.M.F. of phase 1 − 1′, that in line 2 will be in phase with E.M.F. in phase 2−2′, etc.

Now to get the voltage between lines b and c, Fig. 73, it is evident that the voltage Ob must be subtracted vectorially from the voltage Oc. But vector subtraction is performed by reversing the vector to be subtracted, then performing vector addition. This is shown on the vector diagram, Fig. 73, as Ob' and the voltage between lines b and c will be the resultant of Ob' and Oc, given in the diagram by OB. Now the angle $c\,OB = BOb' = 30°$, hence $OB = Oc\sqrt{3}$. In the same way the voltage between lines c and a is given by the vector OC and between lines a and b

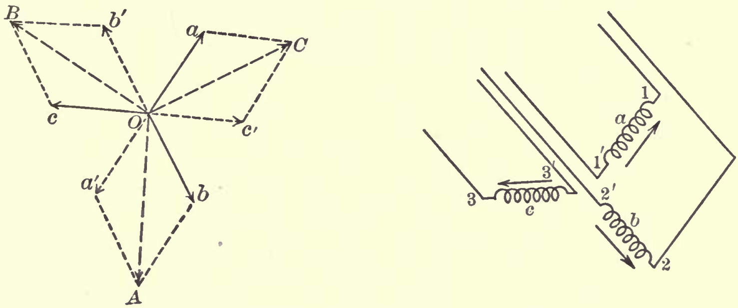


FIG. 73.

is given by the vector OA. We have, therefore, the fact that in the Y connection the line voltages are 120° apart and equal to $\sqrt{3} \times$ (phase voltage). The line current is evidently the same as the phase current.

In the delta connection the relation between phase current and line current may be also determined by vector diagram.

Suppose the phases are not connected, but each is supplying a noninductive load, Fig. 74. Then the currents in the different circuits will be 120° apart as represented by the vectors, Fig. 74 (b). Now if the two lines 1 and 1′ be replaced by a single line the current in this line would evidently be current in (a) minus current in (b). In the vector diagram this is shown by reversing vector Ob to Ob', then adding Oa and Ob' to give the line current OA. In the same way when the lines 2, 2′ are joined and 3, 3′ their respective currents will be given in the vector diagram by OC and OB, respectively. In the delta connection then the line currents are 120° apart and equal to the (phase current) $\times \sqrt{3}$. (Proof same as for E.M.F.'s in Y connection.)

For either connection it is evident that if e and i represent the phase voltage and current respectively, when the loads are balanced, the power being supplied by the three-phase system

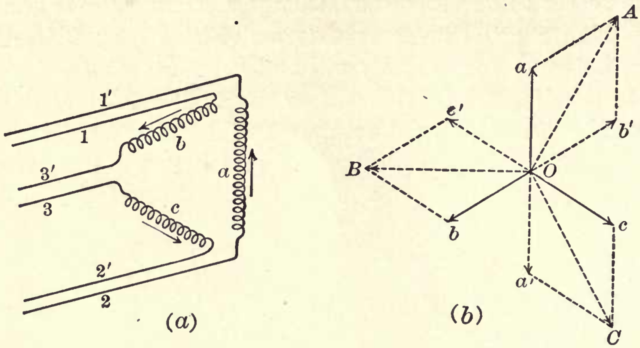


FIG. 74.

$= 3\,ei$ (suppose p.f. $= 1$) or $= 3\,ei \cos\phi$ if the load is inductive. Calling E and I the line E.M.F. and current we have

in star connection $\quad E = e\sqrt{3}, \qquad I = i.$
in delta connection $\quad E = e, \qquad I = i\sqrt{3}.$

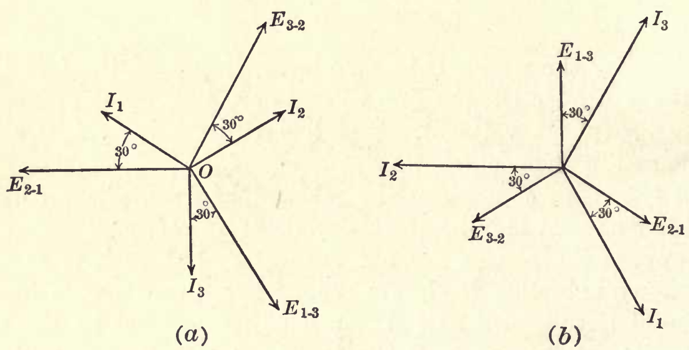


FIG. 75.

Using these values in above expression for power, we have

$$\text{Power supplied by three-phase system} = EI\sqrt{3}\cos\phi,$$

and this expression is the same whether star or delta connections are used.

These relations between E.M.F. and current may now be collected and we may represent vectorially the currents in the

lines and the E.M.F.'s between lines for each connection, first considering noninductive load. Conditions for Y connections are given in Fig. 75 (a) and for delta connection by Fig. 75 (b). If the load is inductive of power factor = cos ϕ, the conditions are represented for the two cases by Fig. 76.

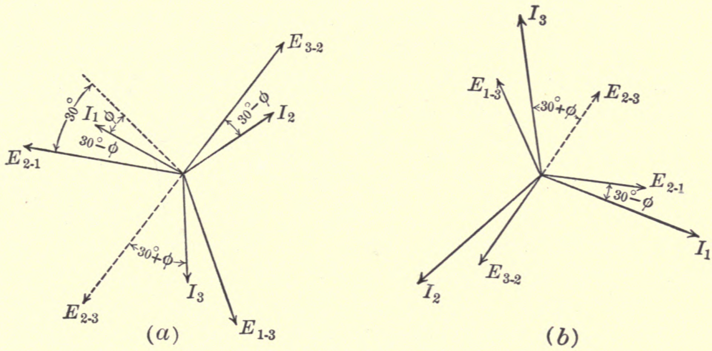


Fig. 76.

Now, if it is desired to measure the power output of the three-phase system, only two wattmeters are necessary as will be shown. Suppose the current coils of the two meters are connected in the lines 1 and 2. The potential coils are to be connected, one end to the line in which the current coil is placed and the two free ends both connected to the third line. The meters will be as represented in Fig. 77. The reading of wattmeter No. 1 will be $I_1\ E_{21}$ cos $(\phi - 30°)$ and wattmeter No. 2 will read $I_3\ E_{23}$ cos $(\phi + 30°)$. In the balanced system $I_1 = I_2 = I_3$ and $E_{12} = E_{23} = E_{31}$, so that

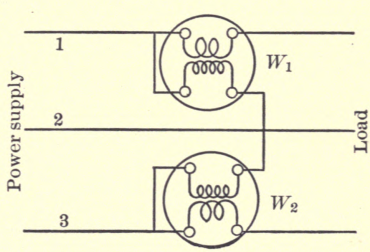


Fig. 77.

$$W_1 + W_2 = EI\{\cos(\phi + 30°) + \cos(\phi - 30°)\} = 2\ EI \cos\phi \cos 30°$$

$$= EI\sqrt{3}\cos\phi.$$

But, as has been shown, $EI\sqrt{3}$ cos ϕ is the power delivered by the three-phase system, hence the sum of the readings

of two wattmeters so connected will give the total power of the three-phase circuit.*

From inspection of the two equations just given it is evident that the power factor of three-phase circuits $= \dfrac{W_1 + W_2}{EI\sqrt{3}}$.

If $\phi = 60°$, then $W_2 = EI \cos 90° = 0$ and if ϕ is greater than $60°$ the meter W_2 will be deflected backwards. Its potential coil must then be reversed and the reading be called negative. If ϕ is such that the current leads (instead of lags) then W_1 reads zero at $\phi = 60°$ and reverses for greater values of ϕ.

For values of ϕ less than $60°$, power $= W_1 + W_2$ and for ϕ greater than $60°$, power $= (W_1 - W_2)$. It is quite evident that if one of the meters is indicating negative power it must be the one having the smaller reading. When doubt exists as to whether or not the power factor of the circuit is less than 0.5 (i.e., $\phi > 60°$) the power factor of the load should be increased somewhat; if the indication of one of the meters decreases, it is recording negatively.

All of the above discussion supposes a balanced load. It can be shown that two wattmeters will measure the power of the three-phase circuit for any condition of balance and power factor. In case the load is unbalanced the term "power factor" loses its significance, as it may be figured out to have several different values according to how the readings of E and I are used and averaged.

If it is known that the load is balanced, then only one wattmeter is necessary. The current coil is connected in one line and one end of the potential coil to the same line. A reading is taken with the free end connected to each of the other lines and the sum of the two readings gives total power.

* Another analysis which shows that the two wattmeters do record all of the three-phase power is as follows:

Let i_1, i_2, i_3, be the instantaneous values of the currents in the three lines.

Let e_1, e_2, e_3, be the instantaneous values of voltage across the three phases of the load, supposing a Y-connected load.

Then we have

$$W_1 = i_1 (e_1 + e_3) = e_1 i_1 + e_3 i_1.$$
$$W_2 = i_2 (e_2 + e_3) = e_2 i_2 + e_3 i_2.$$
$$W_1 + W_2 = e_1 i_1 + e_2 i_2 + e_3 (i_2 + i_1).$$

But, evidently, at any instant we must have $i_1 + i_2 = i_3$ so that $W_1 + W_2 = e_1 i_1 + e_2 i_2 + e_3 i_3$, which is the total three-phase power, at any instant. Hence the two wattmeters record at any instant the total three-phase power, irrespective of power factor or balance.

The power factor of the three-phase circuit may be measured with the use of ammeter, voltmeter and wattmeter; or from the equations derived for the readings of W_1 and W_2 it is seen that

$$\operatorname{Tan} \phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2} \text{ (derive this formula)}$$

so that ammeter and voltmeter are not necessary to find ϕ.

Polyphase circuits are more readily solvable when their quantities are expressed in "equivalent" single-phase quantities. "Equivalent" single-phase current is the value of I, which, multiplied by $E \cos \phi$, gives total power of circuit. In three-phase circuit equivalent single-phase current $I' = \sqrt{3} \times I$. The equivalent single-phase resistance is that quantity, which, multiplied by $(I')^2$, gives total energy used as heat in the system. For either a Y or delta load, R' is equal to one-half the resistance measured between lines.

With lamp banks for load prove the relation of voltage and current in line and phase for Y and delta connections of load. Also by two wattmeters measure the power of the three-phase system and prove (by measuring the watts in the separate phases) that $W_1 + W_2 =$ total power, both for balanced load and unbalanced load.

Obtain a three-phase load (Y connection is the simpler) whose power factor can be made small. Take readings of W_1 and W_2 (also of watts per phase) with ϕ less than 60° and greater than 60° for balanced and unbalanced load. Account for values observed. Calculate the equivalent resistance and current for the inductive load.

To balance a Y-connected load, open phase 3, and, by adjusting for equal voltage drop across phases 1 and 2, balance these two. Then open phase 1 and balance phases 2 and 3, making all adjustment on *phase 3;* then phase 1 may be connected and the load will be balanced.

EXPERIMENT XXVI.

GENERAL POLYPHASE TRANSFORMATION; TWO-PHASE TO THREE-PHASE TRANSFORMATION WITH BALANCED AND UNBALANCED LOAD.

It is impossible to change, by means of static transformers, single-phase power into polyphase power, but having one polyphase system, it is possible, by selecting transformers of proper ratio and properly connecting them, to change to any other polyphase system. In a balanced polyphase system whether two-, three- or six-phase, the power supply is uniform and constant, whereas, in a single-phase system, it pulsates between zero and a maximum. Now, aside from the small amount of energy stored in its magnetic field, it is quite evident that a static transformer cannot act as a reservoir for energy, i.e., cannot take it in at one instant and give it out the next. Hence it is evident, from the standpoint of energy, that single-phase pulsating power cannot be transformed by ordinary static devices into a polyphase system giving off a constant supply of power.

Single-phase power can, however, be changed into a balanced polyphase system by the use of rotating apparatus. In such, the average single-phase power input will equal the constant value of polyphase power output (neglecting the small losses in the machine itself). When the input is in excess of the output the surplus input will be stored in the form of kinetic energy in the rotating member, and will be given out again when the input power falls below the output.

Using exactly the same line of argument, it can be proved that it is impossible to draw single-phase power from a polyphase system and keep the polyphase system balanced when using static transformers. When such a transformation is attempted it will be found that the polyphase system is not evenly loaded, which means that its power supply is pulsating; in fact, the power supply will be just as variable as is the single-phase output.

It has been remarked that any polyphase system, in which the load is balanced, supplies constant, nonpulsating power (the student should prove this point both analytically and geometri-

cally). Such being the case, there is no reason from the standpoint of energy why one polyphase system cannot be changed by static transformers into any other polyphase system. If one system is balanced, its power supply is constant so the other system must also be delivering constant power, i.e., it also must be balanced. If the load on one system is unbalanced it may be imagined as a balanced polyphase load (which is delivering constant power and which must, therefore, be supplied by constant power) and superimposed upon it a single-phase load which utilizes pulsating power and must so be supplied by pulsating power. This component of the total load, i.e., the amount by which it is unbalanced, must be supplied by one phase of the polyphase supply and so the supply system will be unbalanced.

The transformation from polyphase to single-phase power can be accomplished without unbalancing the polyphase system if rotating machines are used. In this transformation the rotating member acts as an energy reservoir so that a pulsating power may be drawn out while constant power is supplied.

It will be shown how any polyphase system can be transformed into any other, with special reference to the two-phase three-phase transformation.

Consider a two-phase supply and two separate transformers connected to the two phases, the secondaries to have a number of taps (as shown in Fig. 78), so that different ratios may be obtained. Let A and B represent the two transformers with extra taps as shown. The quarter-phase star-connected system could be obtained by connecting together taps, 2, 3, 2′ and 3′. If a quarter-phase system of lower voltage is desired the coils may be connected in mesh.

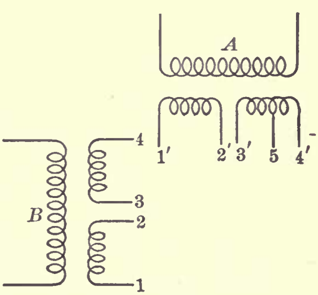


FIG. 78.

To transform a two-phase to three-phase system the T or "Scott" connection is used. On one of the transformers (A in Fig. 79), an extra tap, 5, is brought out. The tap is so connected that the ratio of the voltage between 1 and 4 is to that between 1 and 5 as 1: 0.866. Tap 1′ is connected to the junction of taps 2 and 3 and the three-phase system is obtained from taps 1, 4 and 5 as shown in Fig. 79, where a, b, c is the three-phase line.

That such a connection will give a true three-phase system, the line voltages being equal and 120° out of phase with each other, will be shown by the vector diagram of Fig. 80. The secondary voltages of A and B will be 90° apart but not equal, as there are fewer turns between 1′ and 5 than between 1 and 4. If the voltage 1–4 is taken as 100 volts then voltage 1′–5 = 86.6 volts. The vector diagram of the line voltages is shown in Fig. 80. In plotting the diagram of E.M.F. the actual connection as given in Fig. 79 must be considered, to show whether vectors must be added or subtracted.

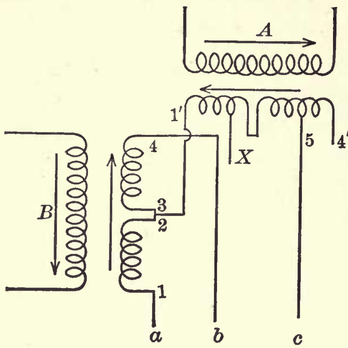


FIG. 79.

The voltage ab is 100 and is put in its proper phase in Fig. 80 directly from Fig. 79. The voltage b–c is the resultant of 4–3 and 3–5. But as both these voltages act toward b, they must be reversed in direction and added to get the voltage bc. The resultant, 4–5, is plotted in the lower part of Fig. 80, as b–c. The magnitude of voltage b–c is equal to $\sqrt{86.6^2 + 50^2} = 100$ volts. Its phase position with respect to ab is $\left(90° + \tan^{-1} \frac{1}{\sqrt{3}}\right) = 120°$.

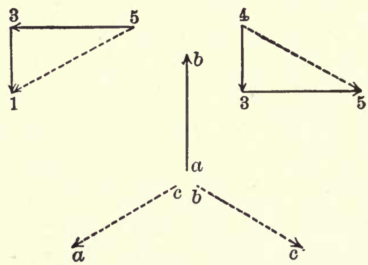


FIG. 80.

Therefore, $bc = ab$, and as vectors they are 120° apart, and bc is so shown in dotted lines in Fig. 80.

The voltage ca is evidently the vector difference of 5–1′ and 3–1, or the vector sum of 5–1′ and 3–1 reversed, and is so constructed in Fig. 80. Its magnitude $= \sqrt{86.6^2 + 50^2} = 100$ volts and its phase angle with respect to bc is 120°, and it is so plotted in dotted lines in Fig. 80. So that this "T" connection of transformers changes two-phase to three-phase power.

The neutral point of this three-phase system may be obtained by bringing out a tap in transformer A, one-third of the way from 1′ to 5 as indicated at X.

The question of power furnished by each transformer is important. If we assume a noninductive load of 10 amperes per line on the three-phase line, the load and volt-ampere readings of the two transformers, on the three-phase side, will be:

	E	I	Cos ϕ	Watts	Volt-amperes
Transformer A	86.6	10	1.00	866	866
Transformer B	100	10	.866	866	1000

From this it is seen that in the Scott connection slightly more transformer capacity is required than in the delta or star three-phase connection, in both of which cases the total volt-amperes for all transformers, for noninductive load, is equal to the watts load, whereas in the T connection the transformers must have about 8 per cent greater capacity.

Any vector may be resolved into two components at right angles to each other. Hence any polyphase system may be obtained from a two-phase system, the only requisite being that the transformer ratios must be certain values and that the sections of the two transformers must be connected in series in right relations to one another (i.e., direct or reversed).

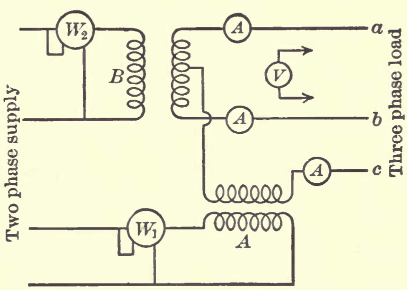


Fig. 81.

With two transformers arranged for Scott connection, connect to a two-phase supply as in Fig. 81. With no load on the three-phase side, measure all necessary voltages to prove that the above given vector construction is correct. The equality of the three-phase voltages may be directly tested by the voltmeter. That the three voltages must be 120° apart may be seen by supposing the three lines connected by equal resistances as in Fig. 82. By voltmeter we measure the magnitude of the voltage AB; then that of BC and then that of CA. But as A cannot have, at any instant, more than one value of potential, so that in traversing the circuit ABC we come back to a point of the same potential as that from

Note. — It is left for the student to prove by vector construction the power factors assumed above for the transformers A and B and also the statement made in regard to the Δ and Y connection of transformers.

which we started, the three voltages AB, BC, and CA, when plotted as a triangle, must close. But the only triangle having sides of equal magnitude is one in which the sides are 120° apart. Of course it must be remembered that if the sides of an equilateral triangle are considered as vectors, plotted *in the same direction* around the triangle, these vectors are 120° apart and not 60° as might be supposed if the *inside* angles of the triangle were considered.

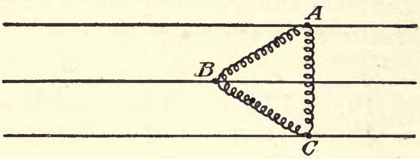


FIG. 82.

Now put a balanced noninductive load on the three-phase side and note whether the two-phase side is balanced. Take similar readings with two conditions of unbalance on the three-phase side. By taking suitable meter readings when the three-phase load is balanced, show that the power factors assumed for transformers A and B in the foregoing discussion are correct. If the three-phase system is only loaded in phase ab how much load will each transformer carry? If only loaded on phase ac or bc how will the two-phase system be loaded? Prove both of these answers by suitable measurements.

EXPERIMENT XXVII.

THREE-PHASE TRANSFORMATION; HIGHER HARMONICS IN THREE-PHASE CIRCUITS.

THE transformation of a three-phase system from one voltage to another may be accomplished by means of one three-phase transformer, also by two or three single-phase transformers.

From the standpoint of first cost and electrical efficiency the one three-phase transformer is to be preferred, but for reliability of operation the three single-phase transformers give better results. The possible connections of the single-phase transformers are

With three transformers:

(1) Primaries, delta	secondaries, delta
(2) Primaries, star	secondaries, star
(3) Primaries, star	secondaries, delta
(4) Primaries, delta	secondaries, star

With two transformers:

(5) Primaries T-connected	secondaries T-connected
(6) Primaries V-connected	secondaries, V-connected
(7) Primaries V-connected	secondaries, Y-connected.

The different methods give different ratios of transformation and also different possible output of transformer groups.

(1) and (2) give the same ratio of transformation, and the possible output of the group is equal to the sum of the ratings of the three transformers because the power factor of **each transformer** is one, on noninductive load.

(3) and (4) also have a group rating equal to the sum of the separate capacities. In (3) the ratio of transformation (for 1:1 transformers) is $\sqrt{3}:1$, while the same transformers connected as in (4) give a ratio of voltage of $1:\sqrt{3}$.

For method (5) the two transformers are connected as in Fig. 83. The ratio of the transformers may be 1:1 and not 1:0.866 as in the Scott two-three-phase transformation. The T connection, however, gives a group capacity only equal to 86.5 per cent that of the sum of the capacities of the individual transformers. This is due to the fact brought out in Experiment 26, that even when the three-phase load is noninductive, one of the transformers furnishes a current of 30° out of phase with its E.M.F.

With a noninductive load the three-phase voltages with the T connection will remain equal, but with inductive load they will become unbalanced owing to the fact that the current in one leg of the transformer A tends to come into phase with the transformer E.M.F. when the load current is lagging, and, as proved in an earlier experiment, a transformer regulates better on noninductive than on inductive load. This effect is diagrammed in Fig. 84, which gives the vector relations of E and I in the two transformers. The vectors a, b, c show the line

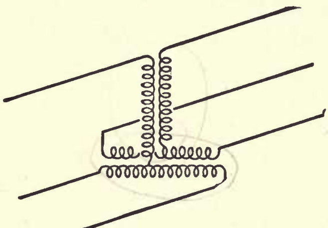


FIG. 83.

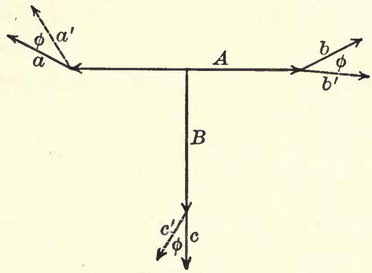


FIG. 84.

currents on the secondary side of the transformers with non-inductive load while a', b', c' show the same quantities when the line currents lag by an angle ϕ.

The half of transformer A which is carrying current a' will regulate worse than the other half because of the greater lag angle of its current. It is to be noted that B will have less than rated core loss.

Method (6) is very frequently employed where an early increase in power consumption is expected. If, e.g., the present load is 2000 K.V.A. and it is expected to rise to 3000 K.V.A., then it is likely that two 1000 K.V.A. transformers will be installed and connected in V. Then when the capacity is to be increased the third 1000 K.V.A. transformer will be added, completing the delta installation. With the two 1000 K.V.A. transformers in V the group capacity will not be 2000 K.W. because of the phase relations of E and I in the transformers. Their combined capacity will be about 1730 K.W. and adding the third 1000 K.V.A. unit will raise the group capacity to 3000 K.W.

Method (7) does not give a balanced three-phase system on the secondary and so is seldom used. It gives what is called the "unsymmetrical Y."

In connecting up for method (6) it is quite likely that the

secondaries may be so connected that they are in opposition (Y) instead of delta and for this reason, and also to investigate the vector relations in the circuit, method (7) will be tried.

The question of upper harmonics in the E.M.F. wave of an alternator becomes of importance in connecting the three-phases of the alternator if the machine is delta wound, or if the phases of the alternator are connected in Y, then the upper harmonics may cause trouble in connecting transformers to the line and so the question will be investigated at this point.

If there were any even harmonics in the E.M.F. wave of an alternator the two halves of the wave would not possess "mirror symmetry," i.e., if the negative loop were moved backward 180° the two loops would not be symmetrical with respect to the time axis. This is shown in Fig. 85, which illustrates a wave containing no even harmonics because the loop b' (b moved back 180°) is symmetrical with a, when the time axis is used as reference line.

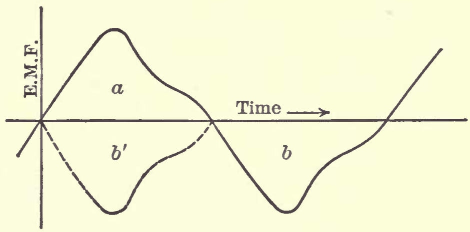


FIG. 85.

That an **alternator** can generate only such waves becomes self-evident when the process of E.M.F. generation is considered. If the winding of a single-phase alternator be developed as in Fig. 86, it is seen to be made up of sets of inductors, a, b, c, in series with each other and these sets of inductors are exactly similar. Hence, whatever E.M.F. is generated at a certain instant will be exactly duplicated 180° later when inductors b occupy the field previously occupied by c, and inductors a are in the same field previously occupied by inductors b. Hence, whatever is taking place at any instant is exactly duplicated (in opposite sense) 180° later. This similarity of the positive and negative loops does not necessitate the loops themselves having a symmetrical form with respect to an axis, through the point of

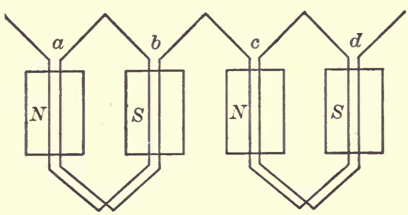


FIG. 86.

maximum value, and, in general, they do not possess this symmetry.

It follows that there are no even harmonics in an E.M.F. wave but there are nearly always some odd harmonics, those of the lower frequencies having the larger amplitude unless some special condition emphasizes one of the higher ones. If the magnetic field was constantly symmetrical, and of proper distribution, with respect to the centers of the poles then no odd harmonics would appear, but armature reaction, inequalities in the air gap, etc., act to prevent this.

The above remarks on generation of upper harmonics have been made upon the assumption that the strength of magnetic field was constant with respect to time, i.e., that the field was possibly distorted, being stronger in one part of the pole face than in another, but that it did not pulsate. If, however, the field is not constant then odd harmonics will be produced by this variation of field strength. In an alternator, having few slots per pole, the reluctance of the magnetic circuit changes considerably with the angular position of the armature because at certain times there will be one more tooth under the pole face than at others. This will be understood by reference to Fig. 87, which supposes a three-phase armature having one coil per phase per pole. Such an armature will cause pulsations in the field of six times the frequency of the armature E.M.F. Calling the frequency of these field changes p, and that of the generated E.M.F. in the armature ω, then there will be superimposed in the frequency ω a harmonic of frequency $(p - \omega)$ which will always be odd, because p is an even multiple of ω.* In a certain machine having a three-phase winding of two coils per pole per phase, the air gap being small, there is a very pronounced eleventh harmonic.†

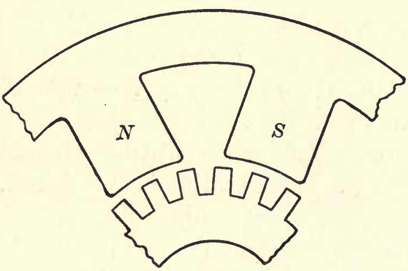


Fig. 87.

* A more complete explanation of the effect of field variations upon wave shapes in the rotating armature will be given shortly. Prof. Pupin has been doing a deal of experimental work on the question and the results will be published soon.

† For curve of E.M.F. generated by this machine see Plate 7 of the Appendix.

The distorted curve showing the magnetizing current of a transformer has for its principle harmonic the third. Undoubtedly all the higher odd ones are present but only the third, fifth and possibly the seventh will have appreciable magnitude. If two harmonic functions are represented by two vectors 120° apart it is evident that the vectors showing the third harmonic of these functions will be directly in phase with one another. Hence, if three transformers are connected in delta to a three-phase line on which there is a marked third harmonic in the wave of E.M.F. their third harmonic currents will be in phase with each other and so will flow in the local path formed by the primaries. This third harmonic will not appear in the line but will do harm in heating the transformers.*

If I = line current, I_t = current in transformer coil and I_3 = amplitude of third harmonic, then

$$\frac{I}{\sqrt{3}} = \sqrt{I_t^2 - I_3^2}.$$

This will only represent approximately the condition as I_3 will include the 9th, 15th, etc., harmonics.

If the primaries of the transformers are connected in Y and the secondaries neither connected in Δ nor loaded, then the voltage wave form across a primary coil will be much distorted. The resulting voltage across any transformer E_t will be greater than $\left(\frac{1}{\sqrt{3}}\right) \times$ line voltage by an amount which equals vectorially the reaction E.M.F. of the third harmonic.

Expressed algebraically $\frac{E}{\sqrt{3}} = \sqrt{E_t^2 - E_3^2}$. This condition results in what is sometimes called the "wabbling neutral"; the voltage from one corner of the Y to the center is constantly changing due to the third harmonic E.M.F.

If a third harmonic E.M.F. is generated in the alternator coils and the coils are connected in delta, then a considerable current may circulate in the armature even when the load is zero. This will cause useless heating of armature. With delta-connected armature, the third harmonic cannot appear in the line E.M.F. (Why?) But in a star-connected alternator the line E.M.F. will contain the third harmonic. If, then, three transformers are

* For curves showing voltage and current forms in three-phase groups of transformers see Appendix, Plates 9–14.

connected in delta to the line a circulating current will flow in their primary coils. In case the primaries are connected star and the secondaries delta, the circulating current will appear in their secondaries. In case everything is connected in star no third-harmonic current will flow unless the neutral points of alternator and apparatus are grounded.*

Make connections and measure E.M.F.'s for all methods of transformation. Obtain sufficient readings that vector diagrams of the E.M.F.'s may be constructed for each connection. By loading with balanced, three-phase, noninductive load and adjusting load until each transformer is operating at rated current, prove that methods 3 and 4 have group capacity equal to sum of separate capacities while methods 5 and 6 give only $56\frac{2}{3}$ per cent the capacity of a group of three transformers, each of same capacity as one of the two used. With the primaries of three equal transformers in delta and not loaded get sufficient data to calculate the value of the third-harmonic current circulating in the delta due to distorted magnetization curve.

By means of the ondograph get the curve of phase E.M.F. of a three-phase alternator giving a distorted wave form. Connect the phases in delta and measure the circulating current by an ammeter. Get the ondograph curve of this circulating current, and on same sheet get curve of phase E.M.F. for reference. Get the curve of line E.M.F. Connect the phases in star and again get the curve of line E.M.F.

Before making the last connection for a delta circuit, whether alternator or transformer, always measure the voltage across the open terminals. The last connection must not be made until this **voltage is practically zero.** If it is not approximately zero, one of the phases is connected in the circuit 180° out of its proper phase and it must be reversed.

In case the transformers are connected $Y - \Delta$ the open corner of the connection may show quite a large value of voltage due to the adding up of the third-harmonic E.M.F.'s around the Δ. This third-harmonic voltage will produce practically no current in the Δ when it is closed. An unbalanced voltage (third harmonic) of 65 volts on some 3 K.W., 110–220-volt transformers caused a circulating current of less than 1 ampere when the Δ was closed. As soon as the Δ is closed a small

* For more complete analysis of the third harmonic in three-phase circuits see Franklin's "Electric Waves."

third-harmonic current flows in the secondary circuit and the effect of this current is to damp out the third-harmonic E.M.F. in the primary circuit. This point may be investigated by connecting one voltmeter across the line and one across one primary and noting the readings of the two instruments before and after the secondary Δ is closed. Before it is closed the readings of the two voltmeters indicate perhaps a large third-harmonic E.M.F.; after closing the secondary this disappears.

Construct vector diagram of all E.M.F. relations obtained and explain group capacity, etc. Explain curves obtained from the alternator.

Why is a three-phase transformer more efficient electrically than three single-phase transformers of same combined capacity? Why are three single-phase transformers more economical from the standpoint of reliability and maintenance cost?

EXPERIMENT XXVIII.

PHASE CHARACTERISTICS OF A SYNCHRONOUS MOTOR; CAPACITY ACTION ON AN INDUCTIVE LINE, "HUNTING" OF SYNCHRONOUS MACHINES.

If two alternators are operating in parallel and the driving power is taken away from one of them it will continue to run, operating as a motor and drawing its necessary power from the other alternator. Moreover, its speed will be exactly the same as it had for parallel operation as a generator. From this fact it has derived its name; a synchronous motor, being supplied with power of a certain frequency, will always operate as a motor at that speed which it would require as a generator to give the frequency with which it is supplied.

A synchronous motor may be either single phase or polyphase, and to make the discussion more simple a single-phase motor will be considered. The characteristic feature of the synchronous motor, outside of its constant speed, is the fact that it cannot be started by supplying power to its armature through a rheostat, as may be done with other types of A.C. motors and also D.C. motors. (This remark will be qualified later when applied to polyphase motors.) The reason for this lack of starting torque becomes evident when the relation of the armature current and the magnetic field is considered.

The field, being supplied from some D.C. source, is uniform in direction and magnitude. The current circulating through the armature conductors is alternating with the frequency of the supply circuit. It is a fundamental principle that a conductor carrying current, placed in a magnetic field (not parallel to the field), will be acted upon by a force which will tend to move the conductor at right angles to itself and the flux. The direction of this force depends upon the direction of the current and of the field; if either of them is reversed the direction of the force reverses, but if both of them reverse the force on the conductor will be in its original direction.

Now consider the stationary armature of a single-phase

synchronous motor as in Fig. 88. One coil of only two conductors A and B is considered, as all of the winding will act in the same way as one coil. At the instant considered A is under a N pole and current is out of the paper so that the force will tend to move the armature in a clockwise direction. In conductor B the current is in the opposite direction as is also the field flux (into the armature under N and out of it under S) so that the force on B will also tend to move the armature in the clockwise direction; hence, it will begin to turn. Owing to the inertia of the armature it will not have moved very much before the current in the conductors will have reversed. The conductors will still be in practically the same position as shown in Fig. 88, because if a frequency of 60, e.g., is used, it is quite evident that the armature starting from rest could not have moved far in $\frac{1}{120}$ of a second; but if the current in the conductors reverses and they remain in a field of same direction as before then their force will be reversed and the torque will be counterclockwise. So that the motor could not start, *per se*, from standstill because the torque varies in direction with the same frequency as the power supply.

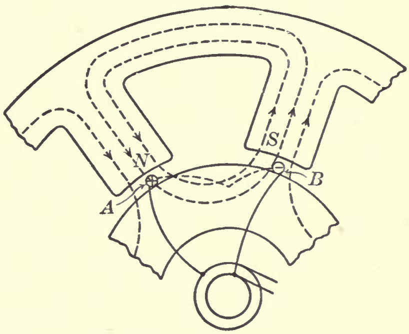


FIG. 88.

But if the armature is revolving rapidly in a clockwise direction at the first instant considered, then by the time the current reverses in direction conductor A will have moved under the S pole and B will be under the next N pole. So in this case the current in the conductor and the direction of the magnetic field in which the conductor is lying both change at the same time and hence the torque will remain in the clockwise direction. If A moves from a N pole to a S pole in the time of one alternation of the current then the armature is revolving at synchronous speed. This discussion gives the reason why a synchronous motor, running at synchronous speed, exerts torque constantly in one direction, and if not at synchronous speed gives a torque alternating in direction, and so is not capable of doing work. So far as torque is concerned a three-phase motor may be considered as

three single-phase motors with this exception, that a polyphase synchronous gives practically uniform torque while in a single-phase motor it varies between zero and a maximum with twice the frequency of the power supply.

In Experiment 15 the conditions to be obtained, before an incoming alternator can be connected in parallel with the line, are enumerated. The synchronous motor must be brought to synchronous speed and these same conditions obtained before it can be connected to the supply line. With some polyphase synchronous motors, however, no extra driver is required to bring them to synchronous speed. A set of transformers with low voltage taps supplies to the stationary armature a polyphase voltage of such a magnitude that the armature current is not excessive (as there is no C.E.M.F. with the armature stationary, normal voltage would cause abnormally large current to flow in armature). The polyphase currents distributed in the polyphase winding of the armature produce a rotating field (to be taken up later on in connection with the induction motor). This field produces eddy currents in the pole faces which react upon the armature so that it is dragged around in the opposite direction to that of its rotating field. It will accelerate, therefore, until nearly synchronous speed is reached.

During this accelerating period there is no direct-current excitation in the field and the field circuit is split up into sections to keep down to a safe limit the induced voltage in the field windings. This voltage may get high enough to puncture the field insulation unless proper caution is exercised in employing this method of starting. When the armature has accelerated up to nearly synchronous speed, normal voltage is applied to the armature (generally by means of a double-throw switch connected to the half voltage taps on one side and full voltage on the other) and the field is gradually excited. The motor will then generally pull into synchronism and proper phase or 180° out of phase; in the latter case it must be forced to slip one pole or the field current may be reversed. In other methods of synchronizing, employing auxiliary power for starting, a lamp or synchronoscope is used; the operation of synchronizing may be done automatically by a synchronizer.

As the field strength and speed of this type of motor are constant the C.E.M.F. must be constant, so that it is not at once evident how the motor can accommodate itself to different loads. In

D.C. motors the C.E.M.F. drops just enough below the impressed E.M.F. to permit the required current to flow. In the synchronous motor, the C.E.M.F. may be very much less than, equal to, or even greater than the impressed E.M.F., and still the motor will operate well for different loads. The explanation of this fact is to be seen by means of the vector diagram of the motor. Under normal operation the C.E.M.F. will be nearly 180° displaced from the phase of the impressed E.M.F. as shown in Fig. 89, where OE_l = impressed voltage, OE_m = counter E.M.F. of motor and OE_r = their resultant. If the impedance of the armature is Z then the current in the armature $= \frac{OE_r}{Z}$ and will lag nearly 90° behind OE_r because of the inductive character of the armature circuit. The output of the motor $= OE_m \times I \cos \alpha = OE_m \times OA$.

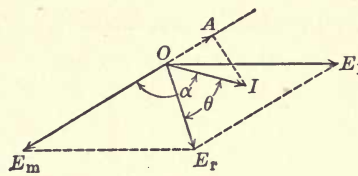


FIG. 89.

If now OE_m remains constant and more load is put on the synchronous motor, the armature will be retarded slightly so that its space phase with respect to the time phase of the impressed E.M.F. is different. It will still be running at synchronous speed, however. This phase displacement of the armature results in E.M.F. relations, as shown in Fig. 90. Here the resultant E.M.F., OE_r, is much larger and hence the current OI is much larger. θ has its same value but α is slightly less. The motor output is again $OE_m \times OI \cos \alpha \doteq OE_m \times OA$, which is about twice as large as its previous value.

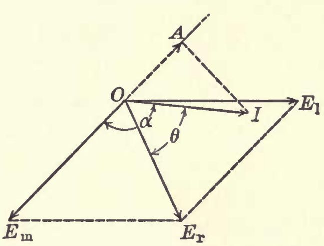


FIG. 90.

If now the load be maintained constant and the field current be varied (thus changing OE_m) OI must so change that $OE_m \times OI \cos \alpha$ remains constant.

In Fig. 91 are given the different conditions under which this might occur. Using the phase of the current as reference line, and assuming a certain value of current OI_1, then the position and magnitude of OE_r are immediately obtained. A circle is described about O with radius equal to line voltage OE_l. If the

motor output $=K$, then we must have $OE_m \times OI \cos \alpha = K$. Draw a line, BC, perpendicular to OI such that $OD \times OI = K$. Then whatever value OE_m may have its projection upon OI must be equal to OD, i.e., we have to construct a parallelogram having OE_r for diagonal, one side terminating on the arc $E_l''E_l'$ and the other on the line BC. Two such constructions are possible, as shown, so that either of two excitations will satisfy the condition. For another value of current and the same load construct the line $B'C'$ perpendicular to OI and so that $OD' \times OI' = K$, and so two more excitations will be found. In such a manner

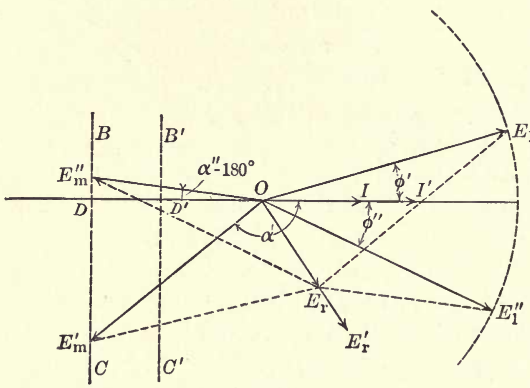


Fig. 91.

it will be found that for any given load on the motor (within its rating) the armature current may vary throughout a wide range, there being two values of excitation for each value of armature current, the greater excitation giving a current leading the impressed force E_l and the other a lagging current. For any load there will be a minimum value of current which will be a singly valued point.

The curve showing the relation between armature current and excitation for any given load is called the " phase characteristic " or " V " curve of the motor. For any given load the most efficient operation will be obtained by adjusting the field excitation to give the minimum value of armature current for that load, because under this condition the armature I^2R loss will be a minimum. If other than this excitation is used the I^2R loss will be greater and also the core loss will be different. For superexcitation the core loss increases and under excitation it decreases. For most efficient operation the total losses (I^2R +

core losses) should be a minimum. This will always occur when the motor is operating close to the bottom of one of the V curves.

It will be noticed in Fig. 91 that overexciting the motor makes the current lead the impressed E.M.F., i.e., the overexcited synchronous motor acts like a condenser, the amount of condenser action depending upon the amount of superexcitation.

As the ordinary transmission line and its load are inductive the current in the line is a lagging one, and so the power-transmitting capacity of the line is less than it should be. A synchronous motor connected to such a line may be made to overcome the inductance of the line and its load so that the line has its maximum capacity. As the excitation of the synchronous motor is increased, first the inductance drop on the line is overcome, and the only drop is the IR drop. If the excitation is further increased the current in the line becomes a leading one and will raise the voltage of the generator by its magnetizing action. Also, if there is a concentrated inductance in the line near the motor, the condenser action of the motor may give what is called a " resonant rise " in voltage, resulting in a higher E.M.F. at the motor terminals than at the beginning of the line. This is resonance in the same way that an actual condenser and inductance resonate.

The " hunting " of a synchronous machine will be briefly taken up here because, in the following tests, hunting of the synchronous motor may occur.

The hunting of a synchronous motor is generally the result of weak synchronizing power; the phase position of the motor oscillates about some proper value of β (for meaning of the terms employed see the discussion on parallel operation of alternators, Experiment 15), this proper value of β depending upon the load the machine is carrying. Now if the change in torque, for a given change of β, is small, then the motor is likely to oscillate violently around this mean value of β, and under some conditions actually pulls itself out of synchronism with the line.

A synchronous motor is, in this action of hunting, analogous to a balance-wheel pendulum. The change in motor torque with variation in β (this change is nearly proportional to change in β) corresponds to the spring action of the pendulum, and the mass of the motor armature corresponds to the mass of the balance wheel.

Now a balance-wheel pendulum has what is called its " natural

period of oscillation" and so has the synchronous motor. If a periodic force of frequency equal to that of the natural period of such a pendulum is impressed upon it the pendulum will oscillate with continually increasing amplitude and the system is said to be in resonance. The amplitude to which the oscillations will build up depends upon the value of the dissipative reactions which are brought into play by the motion — if the friction of the system is low very violent oscillation will result.

In the case of the synchronous motor a similar occurrence may take place; if the load, e.g., is variable and changes in such a way as to result in a periodic fluctuation, the motor will behave as though it was being acted upon by a periodic force. If the period of the fluctuation is near that of the natural oscillation period of the armature the value of the phase displacement angle β will periodically vary and the armature oscillates around its normal value of β, which, of course, depends upon the average load. This oscillation of the armature is called "hunting." It is sometimes so violent that it is impossible to hold a motor synchronized with the line. The amplitude of the hunting of the synchronous motor depends upon the value of the synchronizing force and upon the dissipative reactions which occur due to the oscillations. As the value of β periodically changes, the phase position of the armature reaction, hence the position of the field flux, changes also. Damping grids, so placed in the pole faces that this field fluctuation causes large eddy-current losses in them, tend to keep the hunting of low amplitude, and so practically all apparatus, such as synchronous motors or converters, are equipped with such preventive grids.

If there occurs a large drop in E.M.F. in the line feeding the synchronous machine, so that, with change in current, there occurs a large change in e, the E.M.F. impressed on the machine, the synchronizing effort of the machine is correspondingly reduced as may be seen from the equation given in Experiment 15, which was $\frac{\partial P}{\partial \beta} = \frac{E\,e \sin 2\,\alpha}{2\,R_a}$. This shows the synchronizing force to depend directly upon e. It also depends upon E, the generated E.M.F., of the armature.

A synchronous machine connected to a high impedance line is, therefore, very likely to hunt, and if the hunting is caused by the excessive line drop even damping grids will not remedy the difficulty. The impressed voltage of a synchronous machine

must be nearly independent of the current being taken by the machine if hunting is to be eliminated.

The hunting of a synchronous machine may sometimes be eliminated by changing its natural period; e.g., a small rotary converter hunted so violently that it would not remain in synchronism with the line; a flywheel was mounted on the armature shaft, thus increasing the mass of the oscillating member and so changing its natural period; the hunting after this addition was imperceptible.

With no load on the synchronous motor, and rated voltage and frequency impressed, reduce the excitation until 150 per cent rated current is flowing in the armature. Read impressed volts and frequency, amperes armature and field and watts input to armature. Then increase the field excitation until about the same value of current is obtained leading the impressed E.M.F., and take same readings as before. Get about eight readings in between these two extremes. Get a similar set of readings for half load on the motor and for full load. Take a 25 per cent overload run if the motor will carry it. If the motor has a tendency to " hunt " the readings for low power factors will be difficult to obtain, as under these conditions the tendency to " pull out " is accentuated.

With the synchronous motor running light and at $\cos \phi = 1$, impressed voltage and frequency at rated values, take a reading of terminal volts and field current. Keeping the generator field (i.e., the generator supplying power), constant and the motor still unloaded, increase motor-field current and take another reading. Take about eight similar readings with the field of motor superexcited more each time.

Make another set of readings with a concentrated inductance in series with the line supplying the motor. Read terminal volts, generator volts and field current. In this run the terminal volts may rise above the generator volts due to the " resonant " action of the line and motor.

Try the effect on the operation of the machine of inserting some resistance in series with the supply line.

Plot on one sheet the phase characteristic curves, power factor, and watts input curves and on another the curves of terminal and generator voltages. On both sheets use field current of motor as abscissæ.

EXPERIMENT XXIX.

WITH MOTOR EXCITATION CONSTANT, TO FIND THE RELATION BETWEEN LOAD CURRENT AND POWER FACTOR; PHASE SHIFTING WITH VARIATION OF LOAD.

It is the object of this test to investigate the changes which occur in the armature current of a synchronous motor as the load is increased and the motor C.E.M.F. is held constant; how the phase of the current with respect to the impressed E.M.F. alters with load, and how the phase position of the armature changes as the load is varied.

Referring to Fig. 92, it will be seen that if the impressed voltage and the field current of the motor are held constant, then

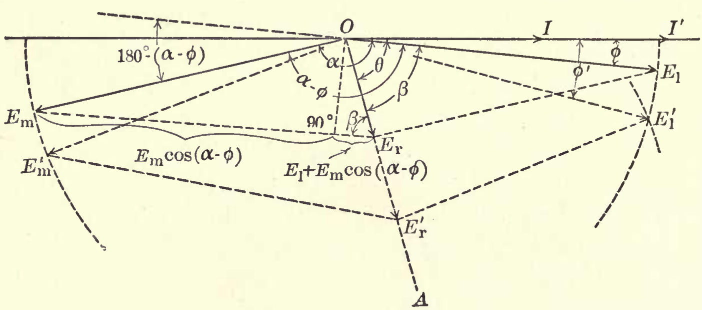


FIG. 92.

the loci of E_l and E_m must be circular arcs about O. The current, OI, will be used as reference line and then the resultant of the motor E.M.F. and line E.M.F. must constantly be on the line OA, displaced θ from OI where $\tan\theta = \frac{2\pi fL}{R}$, L and R being the constants of the motor armature. If Z_m = impedance of motor armature, $OI = \frac{OE_r}{Z_m}$.

If the E.M.F. relations are to be investigated for a certain load K, we shall have the following relations:

$$K = E_m I \cos\alpha \quad \text{and} \quad E_r = IZ_m.$$

Now $$E_r = \sqrt{E_m^2 + E_l^2 + 2\,E_mE_l\cos(\alpha - \phi)}.$$

We have $\beta = (\theta - \phi)$ or $(\alpha - \phi) + (\theta - \beta) = \alpha$,

where $$\cos\beta = \frac{E_l + E_m\cos(\alpha - \phi)}{E_r}.$$

Then

$$K = E_m\left(\frac{E_m^2 + E_l^2 + 2\,E_mE_l\cos(\alpha - \phi)}{Z_m}\right)\cos((\alpha - \phi) + (\theta - \beta)).$$

Now assuming a certain value for $(\alpha - \phi)$ and knowing E_m, E_l and Z_m it is evident that we can obtain the power output, input, efficiency, etc. The power factor of the motor is at once obtained for any value of $(\alpha - \phi)$ by the equation

$$\phi = \theta - \beta \equiv \theta - \cos^{-1}\frac{E_l + E_m\cos(\alpha - \phi)}{E_r}.$$

Although E_l, E_m, θ and $(\alpha - \phi)$ are known it is not readily seen how ϕ will vary, because E_r is itself an involved expression.

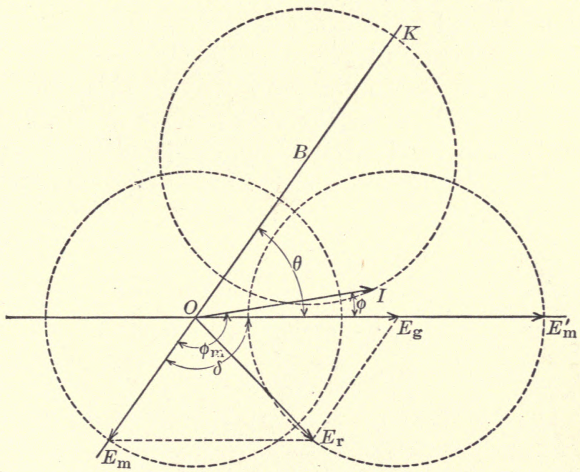


Fig. 93.

Instead of calculating the various quantities derived above they may be predicted by vector construction. Assuming another value of the current $= OI'$, then $OE_r' = OI' \times Z_m$ and with E_r' as center and radius $= OE_m$, cut the locus of E_l at E_l'. The ϕ' is at once obtained and $E_m'I'\cos\alpha$ is also determined.

The application of the circle diagram to the synchronous motor is shown to be possible in "Electric motors," Crocker & Arendt, from which volume it is here reproduced in Fig. 93.

OE_g is taken as reference line. About O is described a circle with radius $= E_m$ and another circle of the same radius is described about the point E_g. This second circle is the locus of E_r. The line OK is drawn at the angle θ with OE_g and $OK = OE_m'$. The center of the current-locus circle is obtained at B by making $BK = OE_m$. The scale for the current is obtained by dividing the volt scale by Z_m. To use this diagram, the angle between E_m and E_g is selected and OE_m is drawn. E_mE_r is parallel to OE_g. With O as center and OE_r as radius cut the current locus at I, and OI represents the magnitude and phase position of the motor current. Then $E_mI \cos \phi_m$ = motor output and $E_gI \cos \phi$ is the value of the input. Cos ϕ is the power factor of the motor and it may be seen from the diagram that the value of ϕ for any load depends upon the relative values of E_m and E_g as was found out in Experiment 28.

Adjust the motor voltage equal to that of the line from which power is to be taken, and synchronize the motor. During the test keep the impressed voltage constant. Read armature current, watts input, watts output, phase position of armature (i.e., angle β, Experiment 15) and impressed voltage. This reading should be taken with no load on the synchronous motor. In case a generator is used for load the connection diagram will be as shown in Fig. 94. The core loss of the D.C. generator must be

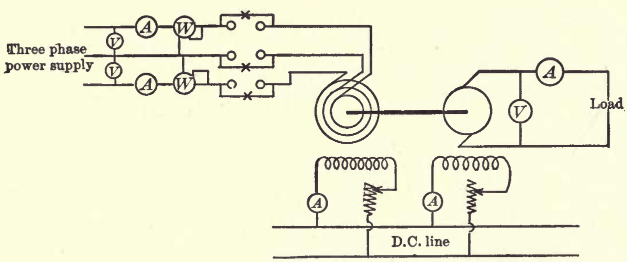


FIG. 94.

measured at a given generated voltage and its field current kept at the value which gives this voltage. The resistance of the generator armature must be measured so that the armature I^2R loss may be calculated.

By means of the D.C. generator (or brake) put on $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, full and $1\frac{1}{4}$ load on the synchronous motor and take same readings as for no load. The field of the synchronous motor is to be kept

constant. Then take a similar set of readings for excitation of the synchronous motor which give motor voltage 25 per cent above and 25 per cent below normal. Measure the impedance and resistance of the motor armature.

Construct three circle diagrams for the three different motor excitations and from them obtain (for such values of the angle ϕ as will give approximately the same motor loads as those actually measured) the values of armature current, ϕ, watts input, and watts output of motor.

Plot curves from the experimentally obtained results of armature current, power factor of motor, phase position of armature and watts input to motor, against watts output as abscissæ. On the same curve sheet plot the same quantities as they are obtained from the circle diagram.

Instead of the circle diagram, the diagram given in Experiment 15 serves excellently to analyze the action of a synchronous motor, with varying excitation and load. The IZ drop in the armature must be reversed in phase to what is shown in Experiment 15, and current phase will be changed, otherwise the construction is the same.

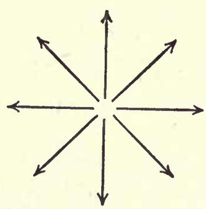
Note. — The circle diagram for the synchronous motor is derived on the supposition of constant impressed E.M.F. and C.E.M.F. In the above test the impressed E.M.F. is maintained constant but how nearly constant the C.E.M.F. of the motor remains is difficult to determine. It is evident that as the phase of the current in the motor armature changes it may have either a magnetizing or demagnetizing action upon the motor field.* In the above test the motor-field current is maintained constant and the effect of the armature reaction is ignored. This may, of course, produce some discrepancy between the actual values of the quantities plotted and their values as predicted from the circle diagram.

* For curves showing how the variation of field strength may change the form of the C.E.M.F. wave see Appendix, Plate 15.

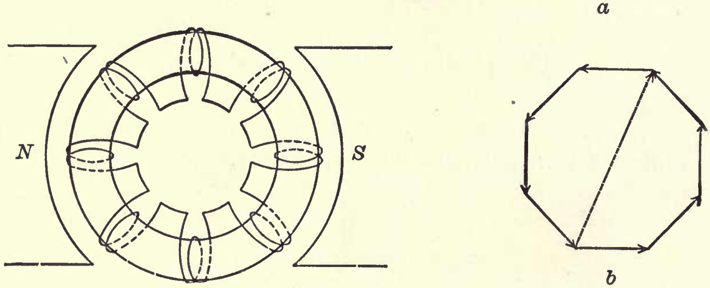
EXPERIMENT XXX.

STUDY OF ROTARY CONVERTER RUNNING FROM THE D.C. END; VOLTAGE RATIOS FOR VARIOUS NUMBER OF PHASES; VARIATION OF VOLTAGE RATIO WITH FIELD STRENGTH; EXTERNAL CHARACTERISTIC FOR INDUCTIVE AND NONINDUCTIVE LOADS; EFFICIENCY.

THE voltage and current relations of a rotary converter can best be studied by supposing that two different windings actually exist on the same armature, the two windings being exactly similar as to number of inductors and their place upon the armature core. One winding is connected to slip rings and the other to a commutator. First a single-phase converter (one having taps 180° apart and which really is a two-phase winding) will be considered. Suppose there are eight coils in each winding and the machine is bipolar. When the armature is rotating each coil will generate a sine wave of E.M.F. (if the field distribution is suitable) and these E.M.F.'s will differ in time phase by the same amount as their respective coils differ in space phase.* The vector representation of these E.M.F.'s



a



b

FIG. 95.

is given in Fig. 95 (*a*). As the coils are connected in series the E.M.F. in the whole armature circuit is zero as shown in Fig.

* If the number of coils is sufficient (say three or more per phase per pair of poles) the E.M.F. relations obtained in the following discussion hold good even

95 (*b*). If, however, the winding is tapped at two connections 180° apart and these taps connected to slip rings it is evident that there will be an alternating E.M.F. on the rings, the maximum value of which is the diagonal of the octagon. This maximum value occurs when the points tapped to the rings are in the neutral plane of the magnetic field. The effective value of this alternating E.M.F. will be $\frac{E_m}{\sqrt{2}}$ where E_m = diagonal of octagon.

Now the second winding is connected to a commutator and the brushes continually connect to the taps which are temporarily in the neutral plane, so that the E.M.F. at the brushes is E_m. If now the armature, instead of being driven by some outside power, is used as a synchronous motor, power being supplied to the collector rings, the ratio of the voltage on the D.C. winding to the impressed E.M.F. will be practically $\sqrt{2}:1$, for when the impedance drop in the A.C. winding is small (which it generally is) the impressed and counter E.M.F.'s are nearly equal.

The current which flows in the A.C. winding will be nearly in phase with the impressed E.M.F., which means that it is 180° out of phase with the vector representing the E.M.F. generated by the group of coils connected in series between taps. The current in each coil will be a sine wave.

If now a current is taken from the D.C. winding the current will flow in phase with the voltage at the brushes (i.e., in phase with the generated voltage of the group of coils on one side of the armature) and this means that it is in phase with the C.E.M.F. of the synchronous motor and hence 180° out of phase with the current in the A.C. winding. Also the current in the D.C. winding is not a sine function but has a constant value for one-half a revolution of the armature, reversing its direction to a negative quantity (of same magnitude as before) just as the coil undergoes commutation.

Now, if the two windings are one, the same winding being con-

though the wave form of the E.M.F. generated in a single coil is widely different from a sine wave. An irregular wave, such as is generally developed in a single coil, signifies the existence of upper harmonics in the E.M.F. wave and these higher harmonics tend ordinarily to neutralize one another when several differently placed coils are connected in series with each other, as they are on a rotary armature. So that with the E.M.F. generated per coil an irregular curve the form of the voltage wave between 180° or 120° taps of the rotary approximates closely to a sine wave.

nected to slip rings on one end and to a commutator on the other, exactly the same relations of E.M.F. and current will be true. The ratio of A.C. to D.C. voltage is as $1:\sqrt{2}$, and in each coil there will be the resultant of a sine curve alternating current and a constant current which reverses its direction once for each alternation of the A.C. current. As this D.C. reverses in the different coils at different times while the A.C. current has the same phase in all coils, it is evident that this resultant current will be different for the different coils. As the rating of a machine depends altogether upon its heating, and the heating upon the current, it is important to consider here more in detail this resultant current. An eight-coil armature will be considered and the different coils numbered for identification as in Fig. 96. The effect of losses in the machine itself will be neglected, so that output = input. The slip-rings are represented outside the commutator and time is reckoned zero when coil 1 is just entering the neutral plane, i.e., the D.C. component of current in No. 1 is just going to reverse. As the A.C. taps are also in the neutral plane, the A.C. C.E.M.F., and hence the A.C. current, are at their maximum values.

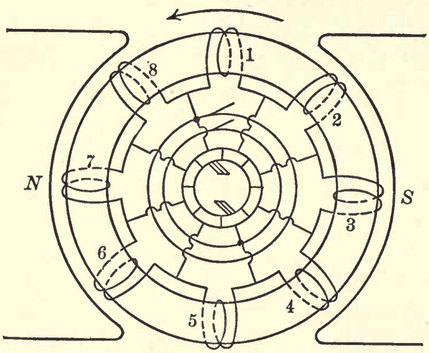


FIG. 96.

Now if the D.C. voltage = 100, the A.C. voltage (effective) = 70.7; and as output = input, if we assume a direct current of 10 amperes, the maximum A.C. current will be 20 amperes. These line currents are twice as large as the coil currents, the armature being two circuit. Then just before commutation the current in coil 1 = 10 − 5 = 5.0 amperes and just after commutation (the A.C. current still being a maximum) the current in the coil = 10 + 5 = 15.0 amperes. After the armature has revolved 45° the A.C. current has a value of 10 × cos 45° = 7.07 amperes, so at this instant current in coil No. 1 (and also 2, 3 and 4) = 7.07 + 5 = 12.07 amperes; but coils 8, 7, 6 and 5 will have 7.07 − 5 = 2.07 amperes at this time. At time = 90° the A.C. current = 0, so that all coils have 5 amperes. By taking

successive intervals of time in this fashion and finding the current in each coil it will be found that the current distribution is not regular, those near the A.C. taps carrying much more current than the others; the form of current wave for two typical coils is shown in Fig. 97.* For this reason the heating of the armature

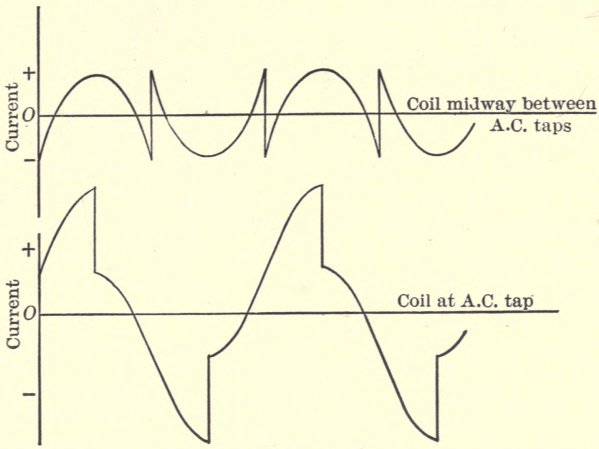


FIG. 97.

of a rotary converter is irregular, those coils next to the A.C. taps becoming the hottest. If, however, the A.C. current is not in phase with the impressed E.M.F. then the hottest coils will not be those adjacent to the A.C. taps but will be more towards the center of the group. Also, for a given load, the actual value of the current in all the coils, and hence the heating, will be greater than where $\cos\phi = 1$.

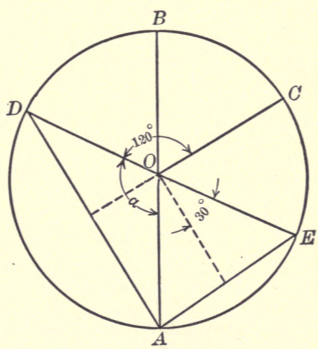


FIG. 98.

If now we consider a polyphase converter instead of a single-phase, the ratio of A.C. to D.C. voltage will evidently be different. For a bipolar machine the D.C. voltage is equal to the vector sum of the E.M.F.'s generated by one-half of the coils as before. But the A.C. voltage will depend upon how many coils are included between taps. If, e.g., a three-phase converter is

* For experimental determination of these current forms see Appendix, Plates 20–23.

considered it is evident that this A.C. voltage will be different than that of a single-phase converter of the same D.C. voltage. For the three-phase machine the A.C. voltage may be found as indicated in Fig. 98. Construct a circle having for diameter $= \frac{1}{\sqrt{2}}$ D.C. volts. This diameter represents the A.C. voltage for a single-phase rotary where the taps are 180° apart. When the taps are 120° apart the effective A.C. voltage between taps is equal to the chord AD, because the circumference represents the circumscribed circle of the polygon formed by the vector E.M.F.'s of all the armature coils, and one-third of the armature coils add (vectorially) their E.M.F.'s to give the voltage AD. If a six-phase converter is used the voltage between adjacent taps is equal to the chord AE.

If α = angle between taps it is clear that

$$\text{A.C. voltage} = \left(\frac{1}{\sqrt{2}} \text{ D.C. voltage}\right) \sin\frac{\alpha}{2}.$$

As the number of taps (i.e., the number of phases) increases, the heating of the armature will decrease due to the fact that the mean resultant current in the different coils decreases with increase of taps. A given rotary having a capacity of 1 used three-phase will have a capacity of 1.44 where used six-phase and a capacity of 0.75 if used as a D.C. generator. (For a more complete discussion of the heating of rotary converters the student is referred to " A.C. Motors " by McAllister.)

As the A.C. voltage bears a constant relation to the D.C. voltage it is seen that with constant impressed D.C. voltage the A.C. voltage cannot be varied by changing the field strength. If, e.g., the field is decreased to ½ normal value the speed of the rotary (running inverted) will increase to twice normal value and so the A.C. voltage will remain constant. If, however, the D.C. voltage is changed, the A.C. will change in the same ratio. It has been said that the ratio of A.C. to D.C. volts is constant, but this is only true when the armature-impedance drop is negligible. As the load on an inverted rotary increases its C.E.M.F. decreases and so the terminal E.M.F. on the A.C. side will decrease.

When the alternating current taken from the rotary is in phase with the generated voltage there will be no appreciable change in field strength due to the armature reaction, because the D.C.

and A.C. reactions just neutralize one another. If, however, the A.C. load is inductive, armature reaction will demagnetize the field and hence the speed must increase to keep the C.E.M.F. on the D.C. end nearly equal to the impressed E.M.F. In some cases this field weakening may be great enough to cause the rotary to run away. This is likely to occur when an induction motor is started with power furnished by an inverted rotary of somewhere near the same size as the motor.

For a given size machine the stray-power and field losses will be approximately the same whether it is used as a D.C. machine, an A.C. machine, or a rotary converter. The armature I^2R losses will be considerably less in the last case, however, so that in general a rotary converter is slightly more efficient than either an A.C. or D.C. generator of the same size.

For a given impressed D.C. voltage, find the A.C. voltage between each and every tap of a single-, three-, quarter- and six-phase rotary converter. Construct a circle having a diameter = $\frac{\text{D.C. volts}}{\sqrt{2}}$ and check the vector diagram given in the previous discussion.

With a fixed D.C. voltage vary the field strength through as wide a range as is permissible and read D.C. volts, A.C. volts and field current and speed. Explain results of these two tests and plot curves to show relations between variables involved.

With constant-rated impressed D.C. volts, and field-current constant at normal value, load the converter on the A.C. side with noninductive load. Read D.C. volts, A.C. volts, field-current, D.C.-current and A.C.-current speed. Take readings at about six different loads between zero and 50 per cent overload. Take a similar run with an inductive load, using wattmeter on A.C. side to get power factor. Maintain power factor constant at about 0.8. Care must be observed that the safe speed is not exceeded.

From data of these two runs calculate efficiency of rotary.

Plot curves of A.C. volts, speed and efficiency, against A.C. current output as abscissa.

EXPERIMENT XXXI.

ROTARY CONVERTER RUNNING FROM A.C. END; STARTING BY VARIOUS METHODS; EXTERNAL CHARACTERISTIC, WITH AND WITHOUT SERIES FIELD ON INDUCTIVE LINE AND NONINDUCTIVE LINE.

There are various methods employed for starting rotary converters and getting them to the proper speed for synchronizing with the A.C. power supply.

If a suitable source of D.C. power is available the rotary may be started from the D.C. end as a shunt motor. Where the rotaries already running are supplying a lighting system the voltage on the D.C. line will be fairly constant and the incoming rotary may be started from the D.C. bus bar. If the D.C. load is railway work the sudden and wide fluctuations in power consumption cause the bus voltage to vary and hence the speed of the incoming rotary being started from such a line will vary and the process of synchronizing is more difficult. Where a station is equipped with a storage battery to carry peak loads this battery is a convenient source of power for starting the rotaries. If the substation is completely shut down the first rotary must be started from storage battery or else one of the methods for starting from the A.C. end may be employed. If several substations are operating in parallel on a distribution system the first rotary in any station may be started from the D.C. end with power furnished by one of the other stations.

Probably the best way to start up a rotary is to have a small polyphase induction motor directly connected upon the shaft of the converter. This motor need only be large enough to furnish the normal stray-power losses of the rotary and must have such a speed-load characteristic that when furnishing power enough to supply the no-load losses of the rotary, it gives the proper speed to make the rotary run in synchronism with the line. This will always necessitate the induction motor having at least one pair of poles less than the rotary. It is impossible to design an induction motor so accurately that its speed, when carrying the load necessary to run with the rotary light, is exactly the

speed demanded by the rotary. If its speed is slightly too high it may be reduced by increasing the resistance of its rotor end rings or by putting a small resistance in two of the leads (on a three-phase motor) supplying its power.

For the process of synchronizing small changes of speed are necessary to get correct phase, etc. These slight changes may be accomplished by altering the field strength of the rotary, thus changing its core losses. This change will result in the rotary not having exactly the same A.C. voltage as the line with which it is to be synchronized, but generally this will do no harm. Its only effect is that the current taken by the rotary upon synchronizing is somewhat larger than it should be. After the rotary has been switched on to the A.C. line the power circuit to the induction motor is opened and its rotor allowed to run free. As there will be no flux in the motor the continuous motion of its rotor, while the converter is operating, requires practically no power.

In using either of the methods so far described a synchronoscope or lamps may be used to determine the proper instant for closing the synchronizing switch. The lamps may be connected for either " dark " or " light " synchronizing, being cross connected for the latter and straight across switch blades for the former. If the lamps used on a polyphase converter do not flicker in phase with one another, one of the phases on the rotary is reversed and a pair of the supply lines must be reversed. Referring to Fig. 99, if the three lamps *a*, *b*, *c* flicker simultaneously the supply line

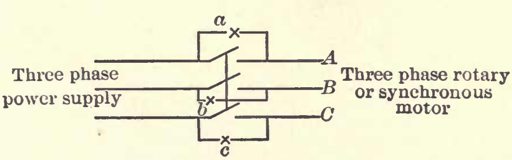


FIG. 99.

is properly connected to the rotary, but if they brighten in rotation, as *a*—*b*—*c*, then two of the leads *A*, *B*, *C*, must be interchanged.

As has been remarked in one of the experiments upon the synchronous motor, a polyphase synchronous machine may be started as an induction machine if its field is left **without excitation** and a reduced voltage is applied to its armature. The current taken is excessive but not enough so to overheat the

machine during the short time necessary for starting. In starting by this method a field break-up switch must be used, otherwise the field coils, acting as the secondary winding of a step-up transformer (the armature coils being the primary), may generate sufficient voltage to break down their insulation.

The recent tendency in rotary converter design has been towards higher voltage machines, 1200 volt rotaries now being used. This high voltage necessitates the use of commutating poles on the rotary, as the voltage per coil is higher than with the 600 volt machine, and therefore commutation is more difficult to accomplish. Now, the use of commutating poles brings about a difficulty when the rotary is started as an induction motor. As the polyphase field, developed by the currents of the rotary armature, passes the commutating poles, owing to the low reluctance of the path of the leakage flux (the face of the commutating pole forming part of the magnetic path) a very heavy current will flow in that coil which is short-circuited by the brushes in the D.C. end of the machine and violent sparking at the brush contacts will result. This sparking is so pronounced in the starting of commutating-pole rotaries by the induction motor principle, that the method cannot be employed unless some special precautions are observed. One type of high-voltage interpole rotary has been designed with a brush-lifting mechanism; while the machine is being started all of the D.C. brushes are lifted from the commutator, then after the machine has reached synchronous speed they are lowered into their proper place on the commutator by a simple lever action.

The induction-motor method of starting will fail if the poles are laminated and no damping grids are used on the pole faces. However, as heavy grids are necessary to prevent hunting of the rotary this method of starting may nearly always be used, but the large starting current necessary disturbs the line voltage to a considerable extent and may even throw out of step other machines already synchronized.

When this method of starting is employed the transformers are provided with low-voltage taps and a double-throw switch is used. The low voltage is applied to the armature until it reaches nearly synchronous speed, when the switch is thrown over and the armature is connected to the power supply of normal voltage, when it will generally pull into synchronous speed. The polarity of the D.C. end of the rotary, as shown by the voltmeter, may, however be reversed; this signifies that the rotary has slipped into

synchronous speed 180° out of its proper phase and some method (as, e.g., restarting) must be used to make it slip back one pole. If the rotary has come into synchronism, in the correct phase, the field current may be gradually increased until such a value is reached as makes the armature current a minimum.

Ordinarily the voltage on the D.C. end of a rotary will fall as the load is increased, for two reasons. The impedance drop of the line increases with increase of load as does also the impedance drop in the armature of the rotary. Hence, if a shunt-wound rotary is connected to an A.C. power line the voltage of which is maintained constant at the generator end, the D.C. voltage of the rotary will fall off considerably as load is applied, the drop being proportional to the armature impedance and line impedance.

As will be readily appreciated, if the line supplying power to the rotary has an extra inductance inserted the drop in D.C. volts will be greater than before, for same values of loads.

So far we have considered a rotary running with constant field strength, i.e., constant C.E.M.F. Now a rotary field may employ series-field excitation as well as shunt-field excitation, in which case the field strength, hence the C.E.M.F. of the rotary, will increase with load. In the discussion of the synchronous motor it was shown that if the C.E.M.F. of a synchronous machine is greater than the E.M.F. of the line supplying its power, then the current taken by such a machine will lead the impressed E.M.F. by a certain angle, the value of which depends upon the amount of superexcitation of the field of the synchronous machine.

A rotary converter having series-field excitation, being supplied with power through an inductive line, tends to automatically compound itself as the load increases; but it has been shown in Experiment 30 that the ratio of D.C. volts to A.C. volts in a converter is constant provided that there is no field distortion (this question of field distribution will be investigated in Experiment 32). Hence it must be that if the D.C. volts of the rotary increase with the load the A.C. voltage impressed must correspondingly increase. This is what actually occurs, and the cause of such rise in A.C. voltage at the end of an inductive transmission line will be seen from the accompanying vector diagrams.

Suppose that the rotary has such field excitation that the current fed into the transmission line is in phase with the generator voltage. Then the E.M.F. relations will be as represented in

Fig. 100, in which OG represents the magnitude of the impressed voltage and the phase of the current. The impedance drop of the line is OZ and the A.C. voltage impressed on the rotary is obtained by subtracting this from OG, giving the rotary voltage OC, which is somewhat less than OG. If now the rotary is much underexcited so that the current in the transmission line lags behind the generator volts the rotary volts will be given by the construction in Fig. 101, in which the letters have the same significance as in Fig. 100. Here it is seen that the rotary voltage is much less than if the current is made to lead the generator volts by over-exciting the rotary; in such case the rotary volts may be larger than the generator volts, as shown in Fig. 102. Now, if the rotary will maintain at all loads a leading current it is evident that the machine will be compounding. In Fig. 103, for example, at no load the A.C. voltage on the rotary is represented by OA. At half load it increases to OB and at full load is OC. If the angle ϕ should decrease with increase of load then the compounding will not be so marked, and if ϕ increases with load, compounding will

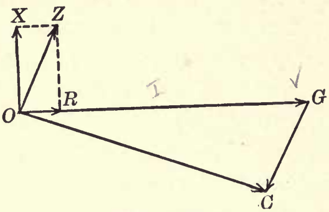


Fig. 100.

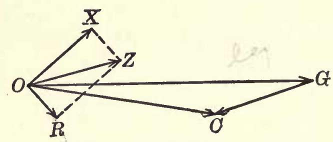


Fig. 101.

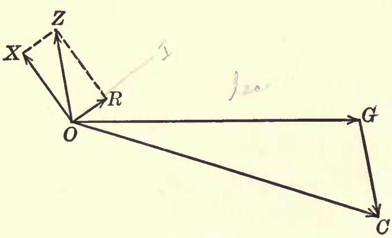


Fig. 102.

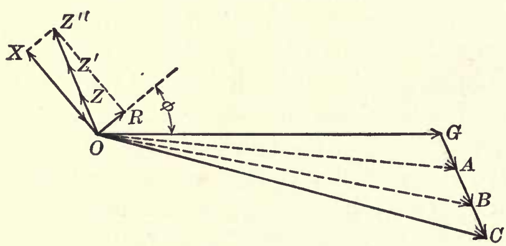


Fig. 103.

exist to a greater extent than if ϕ remains constant from no load to full load. If there are enough turns in the series field the

angle ϕ may be lagging at no load and when the load increases it will gradually change to a leading angle. Under such conditions, compounding will exist to the greatest degree possible.

As may be seen from this discussion and the vector diagrams, for a rotary to be automatically compounding (not considering the synchronous booster) it must be operating on an inductive line and have sufficient series turns to overcome the inductance of the line. The amount of compounding depends upon the amount of inductance in the line and the strength of the series field.

It should be noted that the use of an overexcited rotary for neutralizing lagging current, existing in the transmission line to which the rotary is connected, is not generally advisable. As has been pointed out the heating in the coils of a rotary is very unequal, and although the machine, as a whole, may be running cool enough some few coils may be overheated. This possibility is much exaggerated if the rotary is made to operate at power factors other than one.

In the laboratory to get an inductive line the rotary is supplied through a concentrated inductance. In this case a still greater rise of E.M.F. at the rotary may occur, due to a kind of resonance between the inductance and the superexcited rotary. In this case the terminal voltage of the rotary may become larger than the E.M.F. of the generator supplying power to the rotary.

Try the various methods of starting a rotary; when using lamps to synchronize, try both connections. Explain the effect observed when the phases are incorrectly connected for synchronizing. With constant generator voltage, obtain the external characteristic of the rotary on inductive and on a noninductive line, without the use of the series field. Obtain similar curves when the series field is used to compound the rotary. Keep the shunt-field circuit resistance constant throughout the test, at normal value. For all curves read input (amperes, volts and watts) and output, also generator voltage.

Plot curves of D.C. volts, impressed A.C. volts, power factor and efficiency, using D.C. amperes load as abscissa. If the same rotary is used in this test as in Experiment 30, explain any discrepancy in the efficiency curves of the two tests.

EXPERIMENT XXXII.

STUDY OF THE AUXILIARY POLE ROTARY CONVERTER; VARIATION OF VOLTAGE RATIO WITH DIFFERENT FIELD EXCITATIONS AND EXAMINATION OF FIELD FORM TO CHECK VOLTAGE RATIOS.

In the auxiliary-pole rotary the compounding is obtained by altering the field from its normal distribution. The method can be applied only to three- or six-phase converters, the reason for which will become apparent later.

The D.C. voltage of any generator depends upon the average value of the field under an entire pole face. The exact distribution of the flux is of no moment. Whether the flux density is uniform or not under a pole face will produce no difference in the D.C. voltage generated provided the **average** flux density is the same in both cases.

In the same way, when an A.C. winding is considered with taps 120° apart, the distribution of the flux in the 120° spanned by the coil is not of so much importance in determining the value of the E.M.F. generated. The field distribution will alter the wave form somewhat,* but in a three-phase winding this departure from a sine wave is exceedingly small even when the field has an excessive distortion. The factor which must be observed to maintain constant generated alternating E.M.F.is that the maximum flux which can be embraced by a coil remains constant, i.e., if the flux density is increased in one part of the space covered by a coil, it must be correspondingly decreased in another. Suppose then that the pole of the converter is made in three parts (in which form it was first projected), the excitation of each part being under separate control. Then if all three parts are magnetized equally the field distribution may be represented as by the full-line diagram in Fig. 104. The average value of this field is X and hence X may be taken as a measure of the D.C. voltage generated and also of the A.C. voltage. Now if the part B is weakened and the part A and C both

* This change is very slight as shown by the results given by Plate 27 of the Appendix.

strengthened by an equal amount it will be evident that the total flux under the pole face has been increased and hence the average flux density has been increased. The value of the average density is now given by the line Y and so Y is a measure of the D.C. voltage which would be generated by such a field.

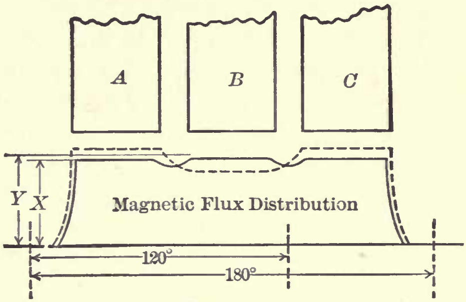


FIG. 104.

But it will be noticed that the average density for that part of the field embraced in 120° of arc (i.e., distance between A.C. taps) is just the same as it was before the field was made non-uniform. This is due to the fact that 120° includes only two of the pole parts and one part is strengthened just as much as the other is weakened. Hence the C.E.M.F. generated between A.C. taps by the machine will be the same with this irregular distribution as it was for uniform field, and as the C.E.M.F. is practically of the same magnitude as the impressed voltage it is seen that a three-phase rotary, designed for a certain normal field, will operate just as well with a distorted field, provided that the distortion introduced does not change the total flux embraced by the windings of one phase and that the distortion is not sufficient to change the form of the C.E.M.F. wave. If such a distortion was introduced that the wave form of the E.M.F. generated was changed, then magnetizing currents would flow from the line into the armature and would immediately neutralize the distorting M.M.F. and bring the field back to its proper form.

The distorted field represented by the dotted line in Fig. 104, however, does change the D.C. volts generated and changes this voltage in the ratio of Y to X. Therefore, it becomes evident that by suitably distorting the field of a rotary the A.C. end will operate as though no distortion occurred, but the D.C. voltage will be raised.

If the middle section of the pole was strengthened and the two outside sections weakened by an equal amount the A.C. voltage would still be the same but the D.C. voltage would be smaller than X.

If the A.C. taps were 180° apart (a two- or four-phase converter) this field-distortion compounding method could not be used because both the magnitude and form of the A.C. wave generated in 180° of the winding will change when the field is distorted as above described.*

To make each field pole in three parts produces a cumbersome and expensive machine. It has been found that the idea of a compounding by field distortion can be carried out with two-part poles practically as well as though three parts were used. One section of the pole is considerably larger than the other. As a two-part pole can be made much smaller and less expensive than a three-part, all split-pole rotaries are at present made with the two-part poles. In the two-part pole construction, however, more difficulty is experienced with the shifting of the commutating plane as the field undergoes distortion. The three-part pole gives practically no shift as the field is distorted. By the use of commutating poles this difficulty of commutation is readily overcome in the two-part pole construction.

Owing to the broken-up character of the magnetic path of this type of converter it will not synchronize readily when started from the A.C. end as an induction motor. Such a converter is very likely to stick at one-third or one-half synchronous speed and not accelerate any more.

The two-part pole and its possible field forms are shown in Fig. 105,† where *A* shows field of main pole alone and *B* and *C*

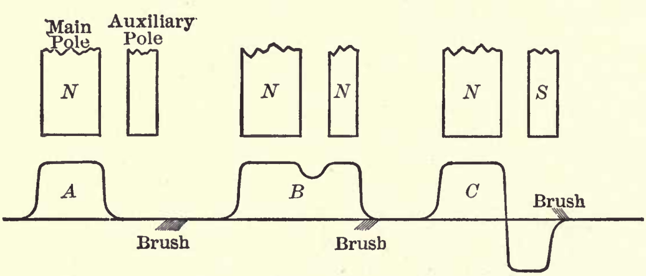


Fig. 105.

show the two forms produced when the auxiliary pole is used; in *B* its current is in the same direction as the main-field current and in *C* it flows in the opposite direction. As will be noticed

* For curves illustrating this point see Appendix, Plate 27.

† For curves showing experimentally determined field forms see Plates 24–26 of the Appendix.

from the figures considerable change may occur in the field at the point of commutation. In this type of converter there is some change in the wave shape of the A.C. E.M.F. when the field is altered. This results in a magnetizing current flowing into the machine from the line and reduces its power factor. In order to keep this power factor as high as possible the main field has to be changed somewhat as the auxiliary field is changed, thus tending to keep the C.E.M.F. wave of the same form and magnitude as that of the impressed E.M.F.

An idea of the corrective effect of the magnetizing current which flows into the A.C. side of a converter when there is a difference in wave form between the generated and impressed E.M.F.'s may be obtained by reference to Fig. 106, in which the

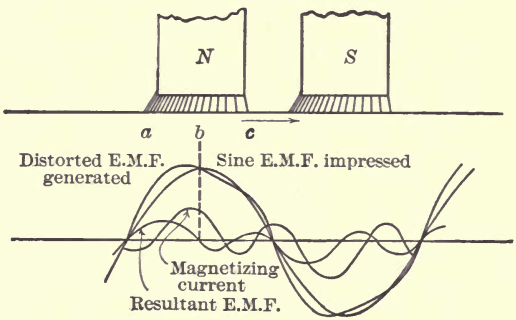


FIG. 106.

field when not subject to armature reaction is supposed to be of such distribution as to give the generated voltage of a form shown by the curve so marked, while the impressed E.M.F. is supposed to be a sine curve shown by the sine curve of Fig. 106.

From *a* to *b* the coil is in too strong a field and the C.E.M.F. is greater than the impressed so that the "magnetizing" current taken from the line during the time *a–b* will be a demagnetizing current. (It was seen in the experiment in the synchronous motor that an overexcited motor draws a leading current, demagnetizing the motor field, and an underexcited motor a lagging current, magnetizing the motor field from the line.) Hence, during the time *a–b* the whole field embraced by the coil will be subjected to a demagnetizing action. At time *B* this magnetizing current falls to zero value because the normal field of the rotary is of the proper value to give a C.E.M.F. equal to the line E.M.F. During time *b–c* the rotary voltage is less than

the impressed E.M.F. and so the magnetizing current will flow in a direction opposite to what it had during the time *a–b*. Hence, while the coil travels from *b* to *c* the rotary field is subjected to the magnetizing action of the armature and so the whole field in the coil becomes stronger, thereby raising the value of the C.E.M.F. as it should be raised during the time *b–c*.

The magnetizing current alternates with a frequency equal to one-half the number of contacts of the two E.M.F. waves. Hence the field of the rotary will fluctuate with the same frequency. When this fluctuation in strength of the rotary field is examined more closely it is seen that it really produces a field which oscillates across the pole face, i.e., at one instant the field is strongest at *a*, and then at *b*, etc. This fluctuation in the field strength has two bad effects. It dissipates energy in the pole faces by the production of eddy currents, and it is likely to cause "hunting" of the rotary. As explained in the analysis of the synchronous motor this hunting may be much lessened by making the field "stiffer," usually accomplished by placing heavy grids of some good conductor in slots in the pole faces. The flux in crossing this grid sets up large eddy currents in the bars of the grid, which currents react upon the field to retard its motion.

In operating a split-pole converter it is desirable to change the current in the auxiliary pole continuously from a positive maximum to a negative maximum. For this purpose a special rheostat with two contact arms is used. The connections of this rheostat are designated in Fig. 107 where *A* and *B* represent the moving arms which travel across the resistance in opposite directions. With *A* at *a* the auxiliary field gets a maximum current in one direction; when *A* and *B* come together in the middle the field has no excitation and then as *A* moves farther towards *b* the current reverses. *A* and *B* are mounted on the same shaft but on oppositely facing dials, so that by the same shaft motion one is moved in a clockwise direction and the other travels in the opposite direction.

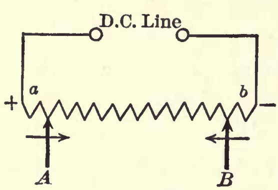


Fig. 107.

The compounding in this type of rotary might be made more or less automatic by using the auxiliary pole to carry a series field, but such is not done on commercial machines.

Make a study of the split-pole rotary and try different methods

of synchronizing; notice any peculiarity in its behavior and explain it. Obtain the external characteristic of the rotary when only the main field is used; adjust the main field before synchronizing so that it gives the proper voltage on the A.C. side, and keep the value of field-circuit resistance at this value during the run. Read amperes, volts and watts input, and amperes and volts output and field current.

With no load on the machine and maintaining constant impressed voltage, take a series of readings to show how the voltage ratio may be changed by means of the auxiliary pole. Begin with maximum negative current in the auxiliary poles and adjust the main field to give maximum power factor (i.e., to make the C.E.M.F. wave as nearly as possible similar to the impressed E.M.F. wave). Read amperes, volts, watts and power factor (with power-factor meter) on A.C. side, volts on D.C. side and both field currents. Note, by sparking at brush contacts, if the field distortion produces shifting of the commutation plane. Get about eight readings in the maximum range of the auxiliary-pole field current.

Make a similar run with full load on the converter, making same adjustments as before and reading the same quantities and also D.C. load current, which is to be maintained constant at rated full-load value. Try one run as above with the commutating poles and one without them and note especially their effect upon commutation.

Running the machine from the D.C. end with constant impressed E.M.F. take ondograph curves of the E.M.F. wave forms between the different taps; by means of a search coil and ondograph get proper curves to show the field distribution under three conditions of the auxiliary-pole excitation (zero and maximum in both directions) and obtain under the same conditions the A.C. voltage generated between 180° taps and 120° taps on the A.C. winding.

Plot curves of external characteristic, efficiency, power factor, and field current, from the results obtained in the run when main field only was used; use load current as abscissæ. Plot on another sheet curves between voltage ratios and auxiliary field current, and on a third sheet curves between main field and auxiliary field current, using auxiliary field as abscissæ. On these two sheets plot curves for both the no-load and full-load runs.

There are in use two other methods for compounding rotary

converters besides the two illustrated in Experiments 31 and 32. One method uses an induction regulator in series with the A.C. supply to the rotary, and the other method employs a synchronous booster on the same shaft as the rotary armature.

A single-phase induction regulator uses the principle of a variable ratio transformer. One coil is connected in shunt with the line supplying the rotary with A.C. power and the other coil, in series with the line supplying power to the rotary, consisting of fewer turns than the first (the number of turns depending upon the amount of compounding desired) is placed at right angles to the first. A movable iron core changes both the direction and amount of flux produced by the first coil which cuts the second, the iron core being in position D (Fig. 108) to assist the impressed E.M.F. and in position D' to crush the impressed E.M.F. The two coils A and B are wound in slots in the laminated yoke C, and the movable core D is manipulated by means of a hand-wheel and worm gear.

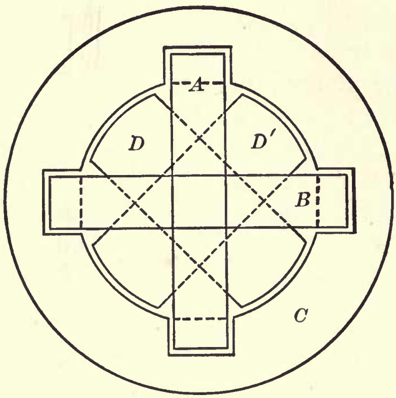


Fig. 108.

The polyphase induction regulator is essentially an induction motor with a wound rotor. The rotor cannot revolve but may be moved (automatically or not, as desired) through an arc of about 180 electrical degrees. The stator winding is connected across the supply line and the rotor winding is put in series with the power supply of the rotary (or vice-versa). The magnitude of the voltage generated by the rotor coils is constant (as a polyphase magnetic field is practically constant in strength) but the phase in which this

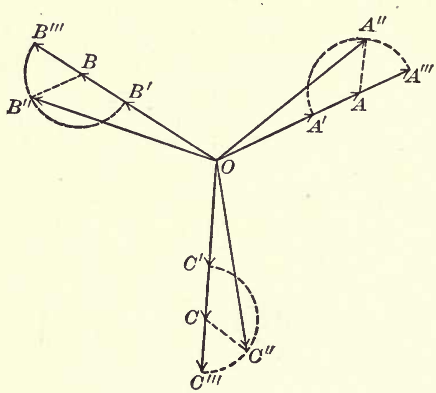


Fig. 109.

rotor voltage is combined with the line voltage may be altered by turning the rotor in different angular positions. In Fig. 109, OA, OB and OC represent the supply voltages and AA', BB' and CC', the voltage generated in the rotor coils. Accordingly as these E.M.F.'s are added to the line E.M.F. in phase, 180° out of phase, 90° out of phase, etc., the rotary will be supplied with voltage OA', OA'' OA''', etc.

In the synchronous-booster system a machine of the same number of poles as the rotary is placed on the same shaft with the rotary; the armature windings of this machine connect to slip rings on one side and on the other connect to the A.C. taps of the rotary. The field of this synchronous booster is adjustable and the supply voltage to the rotary may be changed by varying this field, the maximum range being from full negative field of the booster to full positive field. If the rotary is to be compounded automatically the booster field may be wound in two coils, one carrying a field current which crushes the line voltage and remains constant, the other coil will be in series with the rotary load and has, at full load on rotary, twice as many ampere turns as the other coil. By this means the effect of the booster is automatically changed from a negative E.M.F. to a positive E.M.F. as the rotary load increases.

At no compounding the booster is more efficient than the induction regulator because it has no core losses, while the regulator core losses are independent of the amount of compounding obtained by its use.

If time permits make a study of the induction regulator, polyphase and single phase, also run a test on a booster compounded rotary.

EXPERIMENT XXXIII.

STUDY OF THE INDUCTION MOTOR; OBTAINING ITS CHARACTERISTICS BY LOADING WITH PRONY BRAKE OR GENERATOR.

THE production of the rotating field in an induction motor by polyphase currents and windings has been so thoroughly discussed in various text-books that it will here be taken as understood.* Although the field in such motors is not exactly constant it is so nearly of uniform strength that for test purposes it may be considered so.

In the squirrel-cage rotor the conductors are short-circuited through end rings mounted on the rotor; in the wound rotor the ends of the rotor windings are connected to slip rings, which rings may be either short-circuited or connected together through suitable resistances. In the squirrel-cage rotor the resistance of the paths may be made very low as the winding generally consists of only one bar per slot, but in the wound rotor the resistance of the paths cannot be made extremely low as the brushes and brush contacts offer considerable resistance, however small the resistance of the winding may be.

The currents in the rotor and stator hold the same relations to one another as in a static transformer and it is therefore evident that if but few turns are used in the rotor its current must be correspondingly large; as such large currents would demand heavy rings and brush rigging and would dissipate a lot of energy in the form of heat at contact points, it is customary to put in the rotor winding a number of conductors much greater than in the case of squirrel-cage rotors. In two sample motors, e.g., the rotor has one-third and one-fourth as many turns as the stator winding. The rotor and stator are not necessarily wound with the same number of phases.

The torque of a polyphase induction motor is given by the equation:

$$T = \frac{N_2{}^2 e^2 r_2 s}{\omega\,(r_2{}^2 + s^2\, x_2{}^2)}. \qquad (1)\dagger$$

* See Crocker and Arendt, "Electric Motors," page 180.

† See Crocker and Arendt, "Electric Motors," page 165, et seq.

Where N_2 = number of turns per secondary circuit.
e = induced volts per turn of secondary circuit when $s = 1$.
r_2 = resistance per secondary circuit.
x_2 = reactance per secondary circuit when $s = 1$.
s = rotor slip.
ω = speed of rotating field.

This torque evidently depends upon r_2 and it is found to be a maximum when $r_2 = sx_2$. This condition is obtained by differentiating the torque equation with respect to r_2 and placing its coefficient equal to zero.

The physical significance of this condition for maximum torque may be obtained by analyzing the relation between the current and field which produce the torque. The field of an induction motor is generally such that its space distribution around the periphery of the rotor may be represented by the equation $\phi = \phi_m \cos \alpha$, where α is the angle (in electrical degrees) between the point considered and the axis of the field, as shown in Fig. 110.

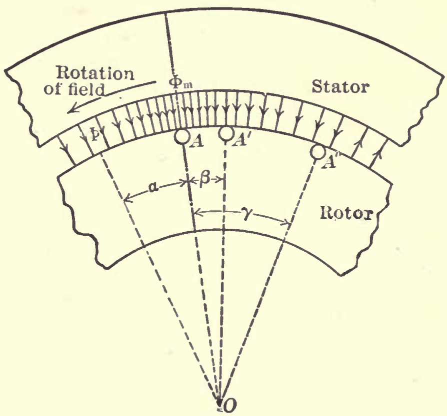


FIG. 110.

Although the following analysis may be readily carried out for any value of slip, it becomes simpler when it is assumed that $s = 1$, and such value of s is here used. The rotor being stationary and the field rotating with a velocity of ω in the direction shown, there will be produced in conductor A an E.M.F. which has a frequency of $\frac{\omega}{2\pi}$, and has its maximum value when the conductor occupies the most dense part of the field, as shown in Fig. 110 at A.

If A is part of a rotor coil of very high resistance the current in A will be in phase with the generated E.M.F. and will have its maximum value when the conductor occupies the position shown at A. If the reactance of the coil is very high the current will reach its maximum value when the E.M.F. has fallen to zero; for

maximum current value the conductor will have the position shown by A'', as here the conductor occupies a field of zero intensity, where the induced E.M.F. is evidently zero. In Fig. 110 it is assumed that the rotor has turned backward 90°. (We assumed the rotor to be stationary and the field to move, but it is easier in the diagram to show the relative motion by moving the conductor from A to A''.) But whenever either the resistance of the coil or the reactance is very large no torque is produced, as may be seen from Equation (2).

If i_2 = value of current in conductor when it is situated in a field of density ϕ, then

$$\text{torque} = K\phi i_2, \tag{2}$$

where K is a constant depending upon length of conductor, diameter of rotor and units in which I_2 and ϕ are measured.

If E_m = maximum value of E.M.F. induced in coil.

I_m = maximum value of i_2.

$$i_2 = I_m \cos(\omega t - \theta) = \frac{E_m}{\sqrt{r_2^2 + \overline{sx_2}^2}} \cos(\omega t - \theta)$$

$$= \frac{E_m}{sx_2} \cos(\omega t - \theta) \sin\theta,$$

in which $t = o$ when conductor is in magnetic axis (as at A) and

$$\theta = \tan^{-1}\left(\frac{sx_2}{r_2}\right)$$

where sx_2 and r_2 are the rotor reactance and resistance. Suppose that at the instant considered the conductor is as shown at A'. Then

$$\text{torque} = T = KI_m\phi_m \cos\beta \cos(\omega t - \theta),$$

but as

$$\beta = \omega t, \quad T = KI_m\phi_m \cos\omega t \cos(\omega t - \theta).$$

The torque produced thus by a single conductor is a double-frequency quantity but it can be shown that the resultant torque of all the conductors on the rotor is a constant quantity, the value of which depends upon the maximum value of the torque produced by one conductor.

Now the maximum value of the torque produced by any conductor depends upon the maximum value of the current in the conductor and the intensity of the field in which the conductor is situated when the current reaches its maximum value.

The maximum current in a coil

$$I_m = \frac{E_m}{\sqrt{r_2^2 + \overline{sx_2}^2}} = \frac{E_m}{sx_2} \sin \theta.$$

The strength of field in which the conductor lies when this maximum current occurs is $\phi = \phi_m \cos \beta$, but it will be noticed that the time phase of the current = space phase of field, so that $\beta = \theta$, hence: torque when maximum current occurs

$$T = \frac{E_m}{sx_2} \phi_m \sin \beta \cos \beta.$$

To get condition for maximum torque, we put

$$\frac{\partial T}{\partial \beta} = \frac{E_m}{sx_2} \phi_m (\cos^2 \beta - \sin^2 \beta) = o,$$

or $\quad \cos^2 \beta = \sin^2 \beta$ which means that $\beta = 45°$.

Hence, for a given inductance of the rotor coil the torque will be greatest when the resistance of the coil is so adjusted that the angle of lag of current in the coil is 45°. But this occurs when $sx_2 = r_2$.

The above discussion is carried out in the assumption that $s = 1$, as we supposed the rotor stationary. The same result will, however, be obtained whatever value is assumed for the slip.

To get a high starting torque the resistance r_2 should, therefore, be made equal to the standstill reactance of the rotor; but such a high value of resistance will give a very small torque near synchronous speed because here sx_2 is small and r_2 should be correspondingly small. Of two motors having the same stray-power losses, the one having the smaller slip for a given load will have the higher efficiency. It may be shown that if the stray-power and primary I^2R loss are neglected the efficiency of a motor = $(1 - s)$.* Therefore an induction motor should give its rated load with as small a value of slip as is possible.

Using the same symbols as before

$$I_m^2 = \frac{s^2 E_m^2}{r_2^2 + \overline{sx_2}^2}$$

and as E_m is fixed, for a certain value of I_m (as e.g., that required for rated output) as we have

$$\frac{I_m^2}{E_m^2} = K, \text{ a constant, independent of } s.$$

* McAllister, "Alternating Current Motors," page 21.

Putting
$$\frac{s^2}{r_2^2 + sx_2^2} = \frac{I_m^2}{E_m^2} = K^2,$$
we have
$$K^2 = \frac{s^2}{r_2^2 + sx_2^2} \quad \text{or} \quad s^2 = \left(\frac{K^2}{1 - x^2K^2}\right) r_2^2,$$
which gives $s = ar_2$, where $a = \sqrt{\dfrac{K^2}{1 - x^2K^2}}$, a constant.

From this it is evident that if it is desired to keep the slip small, r_2 must be kept small. Hence the conditions for good starting torque and high running efficiency conflict with one another.

The problem is solved commercially by having a wound rotor, generally three phase. The rotor slip rings are connected through a three-phase resistance and a controlling switch by which the amount of external resistance in each phase can be cut down in steps from a maximum at starting to zero at normal speed. For small motors the resistance is sometimes located in the rotor spider and a lever with a sliding sleeve on the shaft takes the place of the control switch used with larger motors.

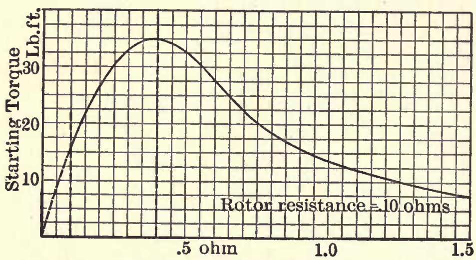


FIG. 111.

The variation of starting torque with rotor circuit-resistance is given in Fig. 111, in which case the maximum starting torque occurs when the resistance of the rotor circuit is 0.4 ohm. Hence, if the rotor circuit itself has a resistance of 0.1 ohm, the proper external resistance to add for maximum starting torque is 0.3 ohm. For a three-phase rotor the circuit would be as shown in Fig. 112, in which the three-phase external resistance and controlling switch are shown. For starting, the switch position is shown by the full

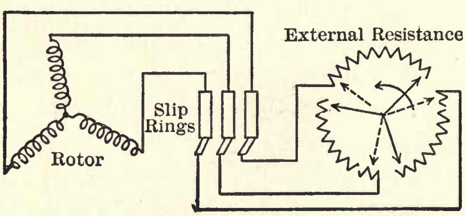


FIG. 112.

lines in the diagram, while for normal running conditions the switch would be in the position shown by the dotted lines.

Besides increasing the starting torque the introduction of extra resistance into the rotor circuit has the effect of cutting down the starting current of the motor. The motor is essentially a short-circuited transformer and even though the reactance of the windings is high (due to leakage flux) still the current taken at standstill may be several times full-load current unless the external resistance is used. The starting-current curve of the motor having external resistance is shown in Fig. 113. The curve $ABCDE$ shows the variations in starting current as the various

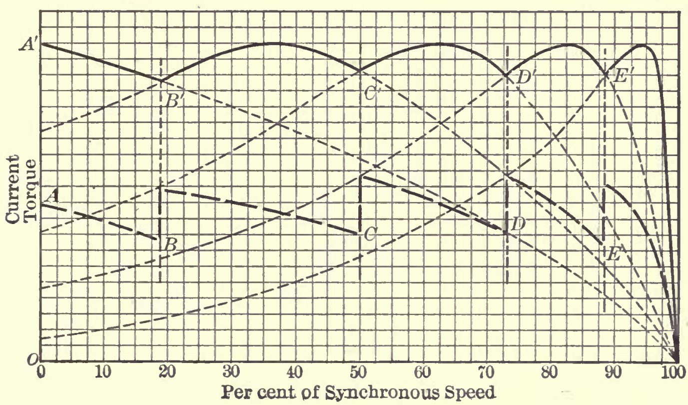


Fig. 113.

steps of extra resistance are cut out with increasing speed and the curve $A'B'C'D'E'$ gives the corresponding speed-torque relation. If the control switch is properly used the motor will operate at nearly maximum torque ($= OA'$) during the whole time of speeding up. The current OA is generally somewhat more than full-load current.

A small motor, equipped with squirrel-cage rotor, gives small starting torque and excessive starting current.

Curves, similar to those of Fig. 113, are given in Fig. 114 for a squirrel-cage motor thrown directly on the line. From a comparison of these two sets of curves the advantage of the wound rotor over the squirrel-cage rotor is at once apparent.

Many times a motor equipped with squirrel-cage rotor is fitted with a double-throw switch and is first thrown on to low-voltage

taps of the supply transformers and not connected directly to the line of normal voltage until approaching synchronous speed. The advantage of low-voltage taps is the decrease in starting current. The starting torque is also decreased by this method of starting.

The direction of rotation of a three-phase motor may be reversed by interchanging two of the supply wires and of the two-phase motor by interchanging the wires of either phase.

The effect of increasing the impressed voltage of an induction motor is to increase its speed for a given torque, increase of "pull-out torque," increased magnetizing current and iron loss and slight increase in efficiency. The increase in magnetizing

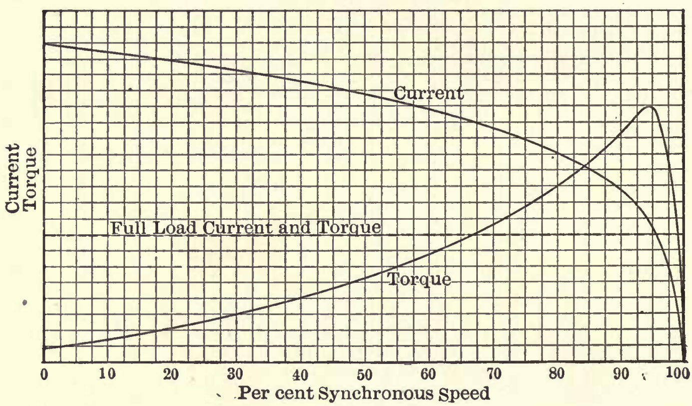


FIG. 114.

current becomes very rapid for an increase of voltage of more than about 20 per cent above rating and results in poor power factor.

A variation in the frequency of the supply gives results similar to those obtained for the transformer, a decrease in frequency causing larger magnetizing current, greater core loss, etc.

Under normal conditions the slip of an induction motor having no extra resistance in its rotor circuit is small and the speed cannot be accurately enough obtained by the ordinary speed counter. Several methods have been devised, however, which permit very exact determinations of the speed. They all work on the idea of obtaining the slip, from which the actual speed can easily be obtained.

The stroboscopic method employs an A.C. arc fed from the supply line of the motor and a disc mounted on the end of the rotor

shaft. This disc is painted alternately black and white in sectors, there being as many white sectors as there are poles on the motor (or some submultiple of the number of poles on the motor).

Suppose a two-pole motor is considered; the disc will be painted in quadrants one black and the next white as in Fig. 115. Suppose the rotor is turning at synchronous speed and that the rotor is in the position shown when maximum E.M.F. occurs, which will be the time when the A.C. arc is most brilliantly illuminating the disc. As sector *a* moves away from the upper position the arc dies down and does not again have its maximum illuminating power for one-half cycle, in which time, if the rotor is turning synchronously, *a* will be in the lower position on the disc and

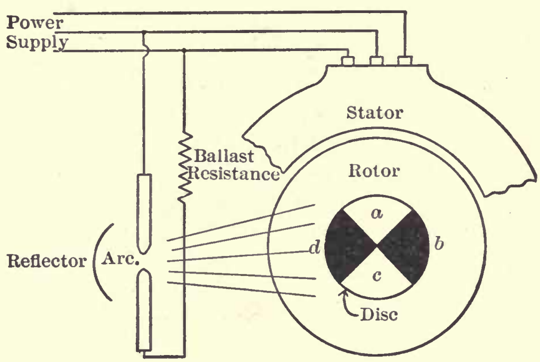


FIG. 115.

white sector *c* will be occupying the position which *a* occupied one-half period previously. The eye is unable to distinguish the difference between sectors *a* and *c* and so gives the impression that sector *a* is stationary in space.

When such a disc is turning at synchronous speed the disc, therefore, appears stationary; but if the rotor has some slip, sector *c* will occupy some position lower than the upper one at the end of one-half period, while sector *a* will occupy a position on the lower side of the disc but not so far advanced as that occupied by *c* one-half period previously. The eye interprets this phenomena by giving the observer the impression that sector *a* (and of course all the other sectors) are moving slowly in a direction **opposite** to the motion of the rotor. The number of apparent revolutions of the disc per minute divided by synchronous speed gives the per cent slip.

Another convenient method, for measuring the slip, consists of lighting an incandescent lamp from the motor's supply circuit through an insulating disc having a conducting strip on part of its periphery. The disc and brush may be mounted on the spindle of a speed counter. Connections for this scheme are shown in Fig. 116. If the disc is turning at synchronous speed the condenser will receive the same charge at every contact so that the lamp will burn with uniform brilliancy, but if the disc is running at less than synchronous speed the condenser receives a varying charge according to the value of the line voltage at the time of contact. The lamp will give a bright period for every time the rotor slips an alternation, i.e., for every 180° (electrical) of slip. Hence, the number of bright periods per minute divided by the number of poles on the motor gives the slip in r.p.m. and the slip in per cent is obtained by dividing this number by the (revolutions indicated on the counter + the number obtained as slip).

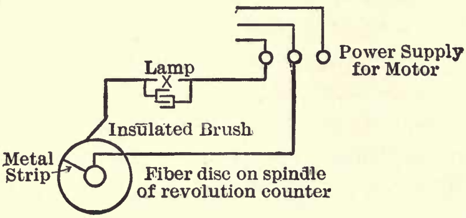


FIG. 116.

If the slip is so great that the number of flickers cannot be accurately counted, the connections as given in Fig. 116 may be modified as shown in Fig. 117. By connecting the D.C. line in series with the A.C. line, the number of flickers per second will be reduced to one-half the previous value. The effect of the D.C. line in series with the A.C. is to make all of the alternations positive (or negative); of course the lamp used must be able to stand 220 volts instead of 110 volts as for connections of Fig. 116. With connections as made in Fig. 117 the formula for obtaining slip becomes

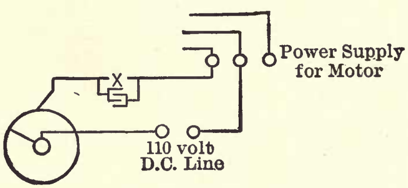


FIG. 117.

$$\text{Slip} = \frac{\text{number of flickers}}{\text{pairs of poles on motor}}.$$

Many modifications of this method will occur to the student, e.g., counting the beats in a telephone receiver properly connected.

There is on the market a slip-meter, from the indications of which, by use of a table sent with the instrument, the per cent slip may be directly obtained.

Study the construction of the three types of motors discussed above. Try the effect upon the rotation of reversing one of the phases; of reversing two of them.

Perform a load run on a polyphase motor, using a prony brake or generator for load. Impress normal voltage and frequency and keep these quantities constant during the run. Measure the current, volts, watts input and torque. Obtain speed by using one of the above-mentioned methods for obtaining slip and calculate synchronous speed from frequency and number of poles. If the number of poles cannot be ascertained by examination run the motor light, under which condition it will run at a speed within a fraction of one per cent of synchronism, from which the number of poles, and hence synchronous speed, can be computed. A very little practice with induction motors is needed to tell immediately the number of poles; e.g., a motor supplied with 60-cycle power running at 875 r.p.m. could not be other than an 8-pole motor. (If external resistance is used in the rotor circuit this method of reasoning will not hold, as it might be a 6-pole motor with a proper resistance in the rotor to cut the speed down from its normal value, which would be between 1150 r.p.m. and 1200 r.p.m.)

Take readings with the motor running light, also at $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, full and $1\frac{1}{4}$ rated load. For setting the brake to its proper value for these loads, the torque for the various loads may be calculated upon the assumption that the motor will maintain synchronous speed. Then adjust the brake to get approximately the value so determined.

Plot curves of speed, efficiency, power factor, current (equivalent single-phase) and torque, with H.P. output as abscissa for all curves.

Note. — If a three-phase motor is used, two wattmeters will be used and at light load one of them is very likely to read negatively. See experiment on "Measurement of three-phase power."

EXPERIMENT XXXIV.

PREDICTION OF INDUCTION-MOTOR CHARACTERISTICS BY THE METHOD OF THE CIRCLE DIAGRAM.

ALL of the characteristics of an induction motor can be predetermined from the "circle diagram," for the construction of which it is necessary to take only two sets of readings from the motor. The application of this circle diagram depends upon the following fact; as the load upon an induction motor increases its rotor circuit may be accurately represented by a resistance and inductance, in series with each other, connected to a constant-potential line, the resistance having different values corresponding to different loads upon the motor, and the inductance remaining constant. The locus of current for such a circuit is a semicircle, as was proved in Experiment 9. As the induction motor is essentially a transformer in its current and E.M.F. relations, any current in the rotor must have an equal and opposite current in the stator. The stator current will be represented, therefore, by the rotor current plus whatever current flows in the stator circuit when the rotor current is zero.

Before the use of the circle diagram is justified it must be shown that the effect of putting mechanical load on the motor has the same effect upon the stator and rotor currents as is produced upon the secondary and primary currents of a transformer when the secondary resistance is varied.

It has been shown previously that for any definite value of rotor current we have $s = ar_2$. Now when $s = 1$ the motor can be doing no mechanical work and so all the energy fed to the rotor must be used up as I^2R loss, as it is in a static transformer feeding lamps; but for any value whatsoever of I_2, r_2 can be so chosen that $s = 1$, ((*a*) involves the values of I_2 considered). Hence the current relations in any induction motor can be accurately represented by considering it as a static transformer the resistance of whose secondary circuit can be altered to correspond to the load on the motor.

As is usually done, the motor will be considered a transformer with ratio of 1:1 because the different quantities are then more easily compared. The circuit of the induction motor may then be represented by the upper diagram in Fig. 118, which is really the same as the lower figure. The r_0 and x_0 paths in the lower figure are of such values that their admittances permit the passage of two currents of such value that they represent accurately the power component and magnetizing component of the primary current in the upper circuit when the secondary is open. The combined current of the paths x_0 and r_0 represents the "running light" current of the induction motor. With the circuit depicted in Fig. 118, this current will decrease as the value of R is changed, owing to the drop through r_1 and x_1. This change in "running light" current actually does occur when the motor is loaded but the change is only a few per cent. As this "running light" current is itself not large compared with full-load current of the motor, the error introduced, by considering the "running light" current (I_0) to be independent of load, is small, and, as it simplifies the problem, it is considered constant when predicting the behavior of an induction motor. The motor circuit is now represented by Fig. 119. In such a circuit

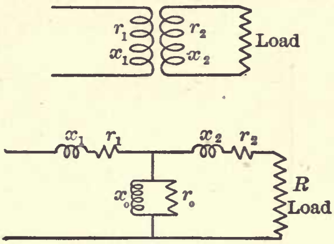


FIG. 118.

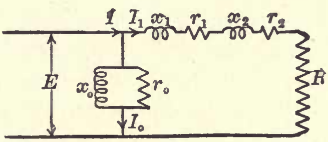


FIG. 119.

$$I_1 = \frac{E}{\sqrt{(x_1 + x_2)^2 + (r_1 + r_2 + R)^2}}$$

and $I = I_1 + I_0$ (vector addition).

In the above equation for I_1, the only variable is R, hence the circuit is equivalent to that of Experiment 9, and the locus of I_1 will be a semicircle of diameter $= \dfrac{E}{x_1 + x_2}$; the locus of I will be the same semicircle but I will be measured from a different origin. All of the quantities discussed here are shown in their proper relations in the diagram shown in Fig. 120.

The measurements necessary for the constructing of this diagram and the method in which they are used are as follows: With normal voltage and frequency, the input in volt-amperes and watts is read with no load on the motor. This is the running light current and by means of the calculated power factor it may be separated into its energy component and magnetizing component. These quantities reduced to equivalent single-phase values are shown in Fig. 120, OZ is the running light current, OP and OQ are its magnetizing and energy components, respectively.

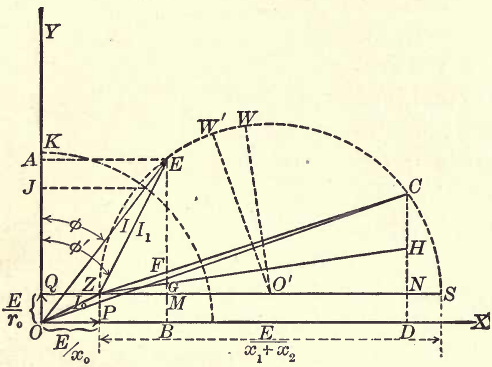


FIG. 120.

The next test to make is the "locked-saturation" curve. This curve is obtained by clamping the short-circuited rotor so that it cannot move, and impressing various voltages upon the stator and reading the corresponding input in watts and amperes. A low voltage is impressed at first and this is increased until about 300 per cent rated current is flowing. About six readings distributed throughout this range will be sufficient. In reading the higher values of current the work must be done very rapidly and the current need not be brought up to its high value until every man who is engaged in the test is ready to read. As soon as a reading has been taken the rotor should be allowed to revolve for a few minutes to give it a chance to cool off.

When the "locked-saturation" curve is plotted it will be found to be practically a straight line, which means that the impedance of the windings is a constant. This is to be expected because normally the iron in the motor is worked at such a low density that even 300 per cent rated current may not saturate the path of the leakage lines. As soon as the teeth become saturated then the impedance will change.

The results obtained from this test are plotted as in Fig. 121 and the curve extrapolated to get current with normal voltage impressed (OA). In the test probably not more than 50 per

cent normal voltage can be impressed without danger of burning the motor. From the value of watts and current input calculate the effective resistance of the motor for the different readings. This should be nearly constant. Use the average value of this resistance and calculate what will be the I^2R loss for current OC. So the power factor of the "locked-saturation" input at normal voltage can be calculated and so the current OC may be separated into its magnetizing and energy components. These currents in equivalent single-phase values are laid off in Fig. 120 at PD and NC, the line ZN being constructed parallel to the X axis; draw the line ZC. This will be a chord of the semicircle and the intersection of its perpendicular bisector with the line QN will give the center of the semicircle at O'. Then the semicircle is constructed with a radius = OZ. The length of the line CN multiplied by rated voltage of motor gives the input with normal voltage and locked motor and so is all I^2R loss. The division of this loss between stator and rotor may be found by measuring the ohmic resistance of the rotor (equivalent single phase) and subtracting this value from the total resistance calculated from the locked-saturation curve results.

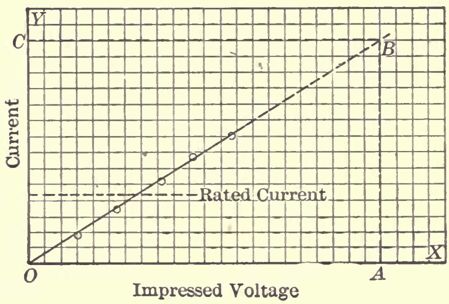


FIG. 121.

Then the point H is so taken that the ratio $\frac{CH}{CN} = \frac{\text{rotor resistance}}{\text{total resistance}}$.

There are several assumptions made in constructing this diagram which make it inaccurate for the prediction of points past maximum output. As the diagram is only used over perhaps 60° of arc these discrepancies do not show themselves appreciably and this method of determining the characteristics of the motor is probably more accurate than by loading.

Note. — It is to be noticed that there is, in this diagram, a peculiar combination of vectors. From the nature of a vector representing an alternating current it is evident that two vectors can be combined to give a resultant vector only when the two have the same frequency. Yet in this diagram the rotor current ZE has a frequency of sf and the stator current has a frequency of f. How then can they be combined as vectors?

At any value of the stator current I, the rotor current is given by I_1, and the running light current by I_0. On the diagram, currents measured along the X axis are wattless and those measured along the Y axis are watt components. (The diagram is constructed for a motor having the same number of turns per circuit in the rotor and stator. If such is not the case the vector addition cannot be directly used.)

The power factor of the motor is given by the ratio of its watt current to total current, or is $\cos \phi$ for the value of primary current $= I$. The rotor power factor $= \cos \phi'$. The power factor of the motor is most readily obtained by constructing a quadrant of a circle with center at O. Have the radius $OK =$ 100 units of the section paper on which the diagram is constructed. Then the power factor is directly obtained by projecting to the Y axis the point of intersection of the primary current with the circular arc. In Fig. 120, e.g., the power factor of the motor when taking current $= I$ is obtained by projecting the point E to the Y axis, as at J, and the value of $\cos \phi$ read directly on the scale of which $OK = 100$.

For the same value of primary current the slip in per cent is obtained by the ratio of $\frac{FG}{EG}$ and motor efficiency is obtained by the ratio of $\frac{EF}{EB}$. The rotor I^2R loss is obtained by the length of FG and the stator copper loss by the length GM. The energy current to supply stray-power losses is given by OQ and the magnetizing current by OP and the leakage current by PB. Torque in pound-feet is given by the expression

$$\frac{EF \times 7.05}{\text{syn. speed } (1 - s)} = \frac{EF \times 7.05}{\text{syn. speed} \left(1 - \frac{FG}{EG}\right)}.*$$

Maximum torque obtainable is found at the point W and is obtained by using above formula for current OW. W is obtained by constructing a radius of the semicircle perpendicular to the line ZH and W', the point of operation for maximum output, is obtained by constructing a radius perpendicular to the line ZC.

Perform Experiment 34 on the same motor as was used for

* The student may derive this expression easily or look it up in some such book as McAllister's "A.C. Motors."

Experiment 33 if possible. Obtain necessary data to construct circle diagram. Construct this diagram carefully upon section paper and obtain from this diagram the following curves: power factor, efficiency, speed, torque, and plot them against H.P. output as abscissæ. Obtain points at about every $\frac{1}{4}$ rated load up to 150 per cent rated load. Predict maximum torque and maximum output of the motor.

Note.—The circle diagram must be very carefully constructed and should have a diameter at least 8 inches.

EXPERIMENT XXXV.

THE VARIABLE-SPEED INDUCTION MOTOR, ITS CHARACTERISTICS BY TEST AND CIRCLE DIAGRAM; VARIATION OF STARTING TORQUE WITH ROTOR RESISTANCE.

WHILE an induction motor is essentially a constant-speed machine, still, for special purposes, it may be used as a variable-speed motor by introducing resistance in the rotor circuit. Its use under such conditions is very inefficient, the method being exactly analogous to the speed control of a shunt D.C. motor by introduction of resistance in its armature circuit. In using the induction motor as a variable-speed device, resistance is introduced into the rotor circuit and hence the rotors of such motors must be wound and attached to slip rings from which the rotor circuit is closed through the rotor rheostat and its controlling switch.

The speed control of an induction motor by this method results in both decreased efficiency and maximum output. It is seen at once that whatever heat is dissipated in the rotor rheostat is so much waste energy, and as the motor has the same maximum torque whatever the resistance of its rotor circuit, any decrease in speed for a given torque must result in corresponding decrease in maximum output. The construction of the circle diagram for a motor having various resistances in its rotor circuit is evidently the same as would be employed if the rotor was short-circuited. The diameter of the semicircle still equals $\frac{E}{x_1 + x_2}$, and this value is independent of r_2. In Fig. 122 the diagram of Fig. 120 is shown and this diagram will hold good for the same motor even when the resistance is added in its rotor circuit. The running light current will still be OZ, the same value it had for short-circuit rotor; but the point C will be changed. If resistance is added to the rotor circuit then the current value ascertained for the locked saturation curve will be less than before and the extrapolated value of locked rotor current under normal impressed voltage will be now only OC', smaller than it was before but still on the same semicircle.

For further increase of resistance in the rotor circuit, the point C moves to C'', giving a still smaller locked rotor current. The maximum torque is fixed by the line WE ($O'W$ being perpendicular to ZB) and it is evident that the rotor resistance does not affect the maximum obtainable torque until the external resistance added is sufficient to bring the point C to W. With this value of rotor resistance, maximum torque is obtained at standstill; any further increase in resistance would require a backward rotation to develop maximum torque.

For a given primary current the torque is evidently the same, i.e., independent of rotor resistance. Also the power factor for a given primary current is independent of rotor resistance. For

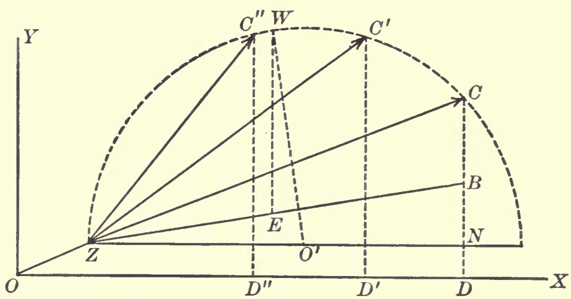


FIG. 122.

a given input (i.e., for a given torque) the slip is directly proportional to the total rotor resistance, while the efficiency and maximum output decrease with increasing resistance. This is exactly analogous to the action of a resistance put in series with the armature of a D.C. shunt motor for speed control.

The effects of added rotor resistance upon the running characteristics of the motor are, decrease of speed directly with the amount of added resistance, decrease of efficiency for a given output and decrease in a maximum possible output. The effect upon the starting characteristics of the motor are, increased torque (until r_2 reaches a value equal to x_2, after which torque again decreases), increased power factor, and decrease in starting current.

In Fig. 123 are given two sets of curves to show the effect of resistance upon the running characteristics of a motor, the full-line curves being for a short-circuited rotor and the dotted curves for rotor with added external resistance.

In Fig. 124 are shown the effects of added rotor resistance upon the starting characteristics of an induction motor.

There are other methods for controlling the speed of induction motors. When used for railway purposes they may be used in a

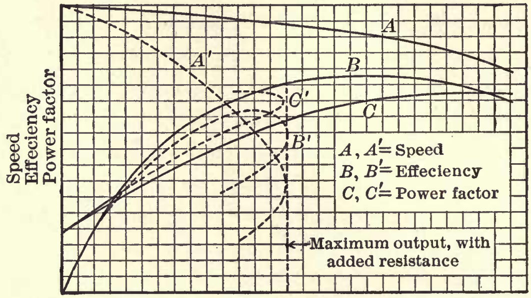


FIG. 123.

kind of series connection, called concatenation, or cascade control. In this method the rotor current of one motor supplies the power for the stator of the second, the rotor of the second being short-circuited or may be used with external resistance. The stator of the first motor is connected to the line.

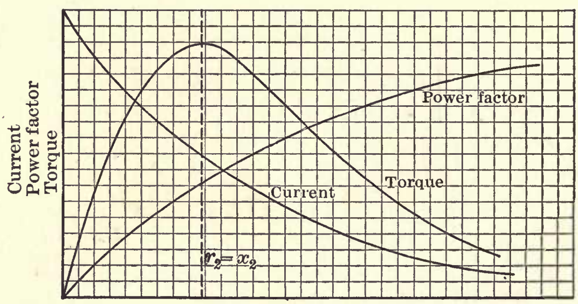


FIG. 124.

The motor may have its stator wound in such a manner that the number of poles is varied by proper switching arrangements. Both of these methods are more efficient than the control by rotor resistance.

The frequency of the power supply to the induction motor may be varied, although this is not a practical scheme commercially, because usually power is distributed at only one frequency.

The voltage supplied to the motor may be varied either by a compensator or an adjustable resistance. Both of these schemes give very poor regulation and are used only in exceptional cases.

Using a prony brake or generator for load, take a series of runs (from no-load to $1\frac{1}{4}$ full-load current) on an induction motor, using different resistances in the rotor circuit for each run. Read volts, amperes and watts input, speed, frequency and torque. Have voltage and frequency at rated values. Take one run with the rotor short-circuited. Take proper measurements for the construction of the circle diagram of the motor under the different conditions.

With about $\frac{1}{2}$ rated voltage impressed in the stator and the rotor locked take a series of readings to show the effect upon the starting characteristics of the motor as the rotor resistance is varied from its maximum value to short-circuited rotor.

Read volts, amperes, watts and torque.

Construct a circle diagram for the prediction of the running characteristics of the motor with its different rotor resistance.

Plot curves (from test results) of efficiency, power factor, speed, torque and primary current for the different resistances. All curves to be plotted with H.P. output as abscissa. Upon these curves indicate (using different color of ink) the points as determined from the circle diagram.

Upon another sheet plot curves of starting torque, starting current and power factor, using total rotor-circuit resistance as abscissa.

All of above curves are to be plotted with equivalent single-phase values of the quantities involved.

If time permits try one of the other methods mentioned for speed control of induction motors.

EXPERIMENT XXXVI.

THE SINGLE-PHASE INDUCTION MOTOR; ITS CHARACTERISTICS; APPLICATION OF CIRCLE DIAGRAM FOR PREDETERMINATION; STARTING AS A REPULSION MOTOR.

THE question of the rotating magnetic field is not quite so readily solved for the single-phase induction motor as it is for the polyphase motor. It is not directly evident how one set of magnetizing coils, supplied with single-phase current, can produce a magnetic field of practically uniform strength, rotating in space. As a matter of fact, the stator coils of such a motor can produce only an alternating field, stationary in space, and it is only by the reactions of the currents in the rotating secondary circuits that a rotating magnetic field is produced.*

After the rotating field has been proved then the treatment of the single-phase induction motor is essentially the same as that of the polyphase motor.

The principle characteristic of the single-phase motor is the absence of starting torque. The reactions of the rotor currents to produce a rotating magnetic field only occur after the rotor has acquired some angular velocity. Such a motor will not start by itself but if the rotor is given a start (and there is no load on the motor) then the motor will accelerate and come up to nearly synchronous speed in the same direction as the rotor is started. The operation of the motor is just the same in whatever direction it revolves, and this is determined by the starting effort.

Compared with the polyphase motor it may be said in general that the power factor, efficiency and pull-out point are all somewhat lower for the single-phase machine, while the speed regulation is somewhat better.

If the stator coils of a polyphase motor produce a rotating field of constant strength, then at synchronous speed there will be no currents in the rotor coils. With the single-phase motor there is a large magnetizing current of double line frequency flowing in the rotor at synchronous speed; owing to this fact there is considerable rotor I^2R loss at all speeds, while in a poly-

* See Crocker and Arendt, " Electric Motors," page 213, et seq.

phase motor this loss is negligible at synchronous speed. At any speed other than synchronous there will be in the rotor a combination of this magnetizing current and an energy component depending upon the load. Hence for given load the rotor I^2R losses are greater in the single-phase than in the polyphase motor. For a given capacity a single-phase motor must have more iron and copper than a polyphase motor, resulting in slightly greater stator core and I^2R losses. These three effects result in giving the polyphase motor a slightly greater efficiency than the single phase.

The decreased " pull-out " point of the single-phase motor is explained by the fact that as the motor slows down the quadrature component of its field decreases and so results in decreased average field strength and so a decreased torque.

As many single-phase induction motors are started as repulsion motors the underlying principle of this type of motor will be briefly discussed. The armature of such a motor is drum wound and connected to a commutator, exactly similar to an ordinary D.C. motor armature; in the repulsion motor, however, the brushes are connected directly together and the armature is not electrically connected to the power supply. Now the E.M.F. and torque generated by such an armature can most easily be discussed by supposing the armature winding to consist of only one turn, this turn being constantly at right angles to the commutating plane. Such an armature is sketched in Fig. 125. Now, if the brushes are so placed on the commutator that the armature winding is represented by the coil (a) it is evident that there will be no current set up in the coil due to the alternation of the magnetic field because the flux does not cut the coil, the flux being parallel to the plane of the coil.

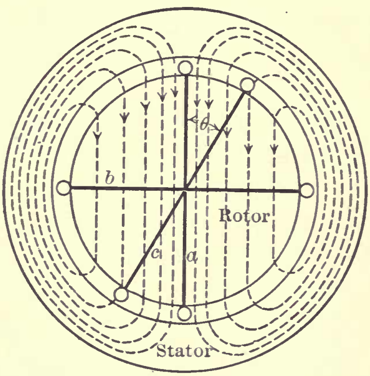


FIG. 125.

If, however, the brushes on the commutator are shifted 90° (electrical), so that the armature winding may be represented by the coil (b) a heavy current will flow because the winding acts like the short-circuited secondary of a transformer. In this position, however, armature can exert no torque because the sides

of the coil b can move only in a direction parallel to the field. Hence with the brushes in either position a or b the rotor will exert no turning effort. Now, if the brushes are set at some intermediate position (so that the armature winding is represented by coil c) a current will flow in the coil and also the coil sides are in a field so that torque will be exerted, tending to turn the armature in, say, clockwise direction. At the next alternation of the field the current in the armature will reverse but the field will also be reversed and so the torque will again be in clockwise direction. The result is a pulsating unidirectional torque and such an A.C. motor gives characteristics very similar to those of the D.C. series motor, giving a heavy starting torque, decreasing its torque and current as the speed increases and having no fixed speed limit.

The magnitudes of the starting torque and current depend upon how much the brushes are shifted from their neutral position, i.e., the angle θ in Fig. 125.

The Wagner single-phase induction motor is equipped with commutator and short-circuited brushes and is started as a repulsion motor, giving a starting torque considerably greater than full-load running torque if so desired. When the motor approaches synchronous speed, a centrifugal device throws off the brushes and short-circuits all of the commutator bars, thereby changing the armature to a squirrel-cage rotor. The machine then operates as a single-phase induction motor.

If the brushes and centrifugal governor are correctly set, the transition from repulsion motor to induction motor takes place without too violent a change in either torque or current. In Fig. 126 in dotted lines are given the speed-torque and speed-current curves of the motor when operating on the repulsion principle and the full lines give the same curves for the motor operating as a single-phase induction motor. The proper speed for the centrifugal switch to act is indicated in Fig. 126; this speed, it will be noticed, is such that the repulsion motor still has somewhat greater than full-load torque when the switch changes the motor to the induction principle. The repulsion motor characteristics may be readily altered by changing the angle at which the brushes are set on the commutator.

Study the construction and operation of a Wagner single-phase motor. Take a load run to get power factor, speed and efficiency as related to the output of the motor.

Try the effect on starting torque and current of changing the

angular position of the brushes (this test may be run at 50 per cent rated voltage so that motor will not overheat). With about 50 per cent rated voltage on the motor terminals obtain its characteristics as a repulsion motor from standstill to that speed at which the brushes throw off.

Obtain sufficient data to construct the circle diagram for this motor. The centrifugal device must be clamped while taking the locked-saturation curve. From the circle diagram get the curves given in Fig. 126 and see how nearly the conditions between repulsion and induction characteristics of the motor tested

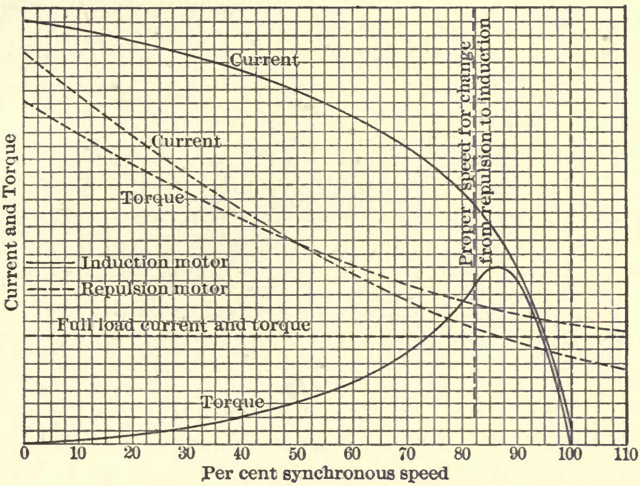


FIG. 126.

agree with the curves given in Fig. 126. If they do not agree what would you do to the motor to make it act as described in the discussion of Fig. 126?

On one sheet of section paper plot curves of efficiency, power factor, speed and current against H.P. output as abscissa, using test results. On another sheet construct the circle diagram and plot the values of primary current obtained from test to see how nearly they lie in the circular locus. To lay off these values lay off the energy component of current up the Y axis of the circle diagram, and through this point draw a line parallel to the X axis. Then with O as center and primary current as radius, intersect this line. Intersection is point desired.

Upon another cross-section sheet plot curves similar to those of Fig. 126 and mark speed at which brushes throw off.

EXPERIMENT XXXVII.

STUDY OF THE INDUCTION GENERATOR; MAGNETIZATION CURVE; EXTERNAL CHARACTERISTIC WHEN EXCITED BY SYNCHRONOUS MOTOR; CHANGE FROM MOTOR ACTION TO GENERATOR ACTION WITH VARIATION OF SPEED, WHEN CONNECTED TO SUPPLY OF CONSTANT FREQUENCY.

If the rotating field produced by the stator winding of a polyphase induction motor is such that its density may be represented by $\phi_m \cos \alpha$, where ϕ_m is the maximum density and α is the angle between the point considered and the axis of the field, then when the rotor is turning at synchronous speed there will be no current flowing in the rotor conductors, as each conductor will continually occupy a position where the magnetic field has a constant value and so the conductor generates no E.M.F. If the stator coils are not so distributed that they produce a field of cosine distribution, this statement is not true; even at synchronous speed currents will flow in the rotor circuits which, by their reaction upon the stator field, transform it into a field of cosine distribution.

With the rotor turning at synchronous speed, the current in the stator will have a magnetizing component of sufficient strength to produce a field which by its change gives a C.E.M.F. practically equal to the impressed E.M.F., and an energy component sufficient to furnish the iron losses of the motor.

Now, no matter what the speed of rotation may be the current relations in the stator and rotor must be the same as exist between the primary and secondary current of a static transformer. When an energy or wattless current is flowing in the secondary the primary must also be carrying an equal current plus its no-load current. Hence, when the rotor of an induction motor is carrying a certain current the stator must be carrying the same current (or rather an equal current in the opposite direction) plus its no-load current (1 : 1 windings considered).

If the rotor of an induction motor is turning at less speed than the magnetic field it is said to have a positive slip. The rotor

conductors, cutting flux, will generate what we will call a positive E.M.F. and there will flow in the rotor circuits a current which will be nearly in phase with the rotor E.M.F. The lag of the current will be $\tan^{-1} = \frac{sx_2}{r_2}$ and for small values of s this angle will be small. This rotor current must have its counterpart in the stator and if the slip is positive this stator current will be such that its product with the impressed E.M.F. represents **power input** to the machine, and it is absorbing electric power and giving out mechanical power, i.e., is running as a motor.

If the rotor is connected to some driving device (e.g., another motor) and is driven at some speed higher than synchronous, it is at once evident that the E.M.F. generated in the rotor conductors will be in the opposite direction to what it previously had. Hence the direction of current in the rotor will be opposite and so must its counterpart in the stator windings. This means, of course, that the current in the stator, due to the rotor currents, is in a direction opposite to what it had when the machine was operating as a motor; the energy component of stator current is, for this condition, flowing in a direction opposite to the impressed E.M.F. Hence, the machine must be **giving off power** and so is a generator. Such a machine is called an induction generator; as the frequency of current delivered is not equal to the rotational frequency of the machine, it is sometimes called the asynchronous generator.

As the slip of an induction machine changes from positive to negative, it has just been shown that the energy component of the stator current changes phase 180° with respect to the impressed E.M.F.

The wattless or magnetizing component of the stator remains in the same phase, however. With respect to the impressed E.M.F. of the motor, this is a lagging current; reckoned from the phase of the motor C.E.M.F. it would, therefore, be a leading current, so that referred to the phase of the generated E.M.F. of the asynchronous generator its magnetizing current is a leading current. For the induction generator to be operative, therefore, it must be connected to a line of certain frequency and some device in the line must be capable of drawing a leading current from the line, the magnitude of this leading current being the amount of magnetizing current required by the generator.

The magnitude of rotor current, and hence of the stator current, depends upon the amount of negative slip. The voltage of the generator depends upon the strength of its field, i.e., upon the amount of magnetizing current furnished to it. These two characteristics are the distinguishing features of the induction generator.

An over-excited synchronous motor has both of the features necessary to make an induction generator operate; it has a definite frequency and draws a leading current. If, therefore, a synchronous motor, with sufficient field excitation, is connected to the terminals of an induction machine of higher rotational frequency than the motor, the induction machine will generate, supplying the amount of power to run the motor, and other load (as e.g., lamps) may be operated from the line.

In starting, the two machines may be electrically connected and both be brought up to a speed somewhere near the proper running speed of the induction machine, then the driving power may be taken from the synchronous machine and it will, if sufficiently excited, continue to run, drawing sufficient power from the generator to supply its stray-power losses and taking from the line a leading current which becomes the magnetizing current for the generator.

As may be seen from this discussion the current relations in the induction generator follow the same laws as in the motor and so the circle diagram must hold good for both if it holds for the motor. In Fig. 127 the application of the circle diagram to the generator is indicated. The no-load current is shown at OP; for motor action some current as OA is required, having an energy component OC and magnetizing component OE. The current OD is required to furnish the no-load losses. If not loaded and not speeded up by a driver the machine operates at perhaps 99.5 per cent synchronism and draws an energy current OD from the line.

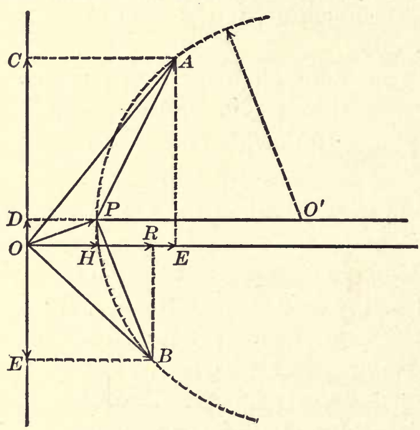


FIG. 127.

As the rotor is speeded up part of the energy required to supply machine losses is supplied by the driver mechanically, and if the speed is increased to perhaps 100.5 per cent synchronism, the stator current becomes OH, purely a magnetizing current, and all of the machine losses are supplied mechanically. From 99.5 per cent to 100.5 * per cent synchronism the machine gives neither mechanical nor electrical power; this is called the region of total loss. At some speed above 100.5 per cent synchronism the energy component of stator current may go to some value as OE, in phase, opposite to OC. The total stator current OB will be found on the same circular locus as OA. The complete application of the circle diagram to induction generator characteristics will be given later.

An understanding of some of the generator characteristics may be obtained by considering what happens when a generator, excited by a floating synchronous motor, is loaded. Suppose that the speed of the generator is adjusted to give a speed of 60 cycles to the synchronous motor and the synchronous moto-field is adjusted to give the induction generator a terminal voltage of 110 volts. If the generator rotor is held at constant speed and some load is connected to the generator terminals two effects will be observed. Both the voltage and frequency of the system will decrease. Even if the generator induced voltage remained constant the terminal voltage would fall with increase of load due to the increased IZ drop in the stator windings. But the induced voltage itself decreases with increase of load. The lag angle of the rotor current is $\tan^{-1} = \frac{sx_2}{r_2}$ and, due to the slowing down of the synchronous motor (explained below), s increases with load so that the rotor currents tend to demagnetize the stator more and more as the load is increased. This may be readily seen from Fig. 127. To give 110 volts induced voltage, e.g., at synchronous speed requires a magnetizing current of OH. When the load on the generator is equal to OE, then the magnetizing current necessary to give 110 volts induced E.M.F. is given by the vector OR. This current is made up of two parts; the original current OH and the demagnetizing component of the rotor current HR. Hence, to maintain constant induced voltage as the load increases the synchronous motor must be

* Of course these numerical values of slip are only approximate; they will be different in different machines.

so excited that at any generator load the motor draws a leading current just equal to this wattless component of the stator current.

The synchronous motor slows down because it requires a certain amount of power to run itself, and if the terminal voltage of the generator falls the C.E.M.F. of the synchronous motor must also decrease to permit the required current to flow. The slowing down of the synchronous motor may be explained from another standpoint. The stator current (load current) can only increase if the rotor current increases, as before noted. The rotor current can only increase if its slip increases and as the rotor is being driven at constant speed its slip can only increase by a slowing down of the field speed. Hence the synchronous motor must slow down sufficiently to make the difference between field speed and rotor speed such that the rotor currents generated produce in the stator just that current demanded by the load.

It will be noticed that this behavior of an induction generator excited by a floating synchronous motor is exactly similar to that of a shunt-wound D.C. generator with its brushes set in such a position that armature reaction demagnetizes the field. As load in such a machine increases the terminal E.M.F. drops because of the increased IR drop in the armature windings and because the induced voltage falls, due to the weakening of the field by the armature reaction. If it is desired to maintain constant terminal voltage on such a machine the shunt field current must be increased sufficiently to overcome both of these effects. In the shunt D.C. machine this is accomplished by cutting out the field rheostat and in the case of the induction generator it is accomplished by increasing the field current of the synchronous motor. The decrease in frequency as the load increases can only be overcome by speeding up the rotor. With increase of load there must be corresponding increase in slip and if the rotor turns at constant speed the synchronous motor (by the speed of which the frequency of the power generated is determined) must slow down.

The magnetization curve of the asynchronous generator can readily be obtained as follows: With the synchronous motor floating on the generator line and no load on line, run the generator at such speed that the motor constantly gives the frequency at which the magnetization curve of the generator is desired. By means of an ammeter, wattmeter and voltmeter

in the line connecting the two machines measure the variation in terminal E.M.F. of generator, watts and current as the field current of the synchronous motor is varied through that range which gives an E.M.F. varying between rated value for the generator and as low as it is possible to go. The generator current may be resolved into its power and wattless components and the relation between wattless, or magnetizing current, and terminal E.M.F. gives approximately the magnetization curve of the generator. The combination becomes nonoperative at low values of voltage so that the foot of the magnetization curve cannot be determined.

An interesting feature of the induction generator is the possibility of excitation by condensers. Although of not much commercial importance at present, an explanation of this method of operation will present the characteristics of the machine from a different standpoint and so will be considered here.

The magnetization curve of the generator will have the general form given in Fig. 128. The impedance of a condenser is $\frac{1}{\omega C}$ and the current in such a condenser is $\frac{E}{\omega C}$. The current is, therefore, a straight line function of the voltage and so may be represented by a straight graph, using current and voltage for the abscissa and ordinate. The equation of the graph will be $I = E \tan \phi$, where $\phi = \tan^{-1}\frac{1}{\omega C}$. In Fig. 128 three such graphs are shown at OC, OD and OE, the graph OC being for a condenser of greater capacity than OD, etc. The magnetization curve of the generator is shown by the curve OAB. If the normal voltage of the generator $= OB'$, then the condenser whose graph is OC will just furnish the proper amount of leading current, OF, to generate the voltage OB'. A smaller condenser

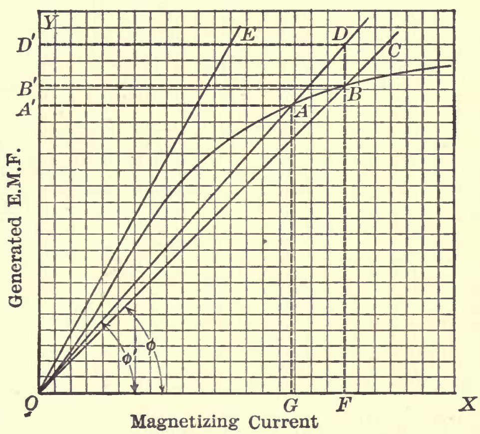


FIG. 128.

whose graph is OD requires a voltage of OD' to draw from the line a leading current of value = OF. But the generator, when furnished with magnetizing current OF, can generate only voltage = OB', which will cause to flow in the condenser a current equal to OG. Hence, the condenser D would not serve to produce in the generator normal voltage. If the generator is operating with condenser C at normal voltage OB', and the condenser D is substituted for C, the voltage of the generator will immediately fall to OA', at which voltage the magnetization curve and condenser graph intersect. With the condenser E the generator could not operate because at no voltage does the condenser draw enough leading current to magnetize the generator to that same voltage; in other words, the magnetization curve and condenser graph do not cross. When once brought up to voltage either of the condensers C and D will serve to maintain the generator voltage, but it is quite likely that if the generator is running idle and either of these condensers should be connected across the generator terminals it would not build up.

If the magnetization curve of Fig. 128 is much magnified and only its lower extremity considered we have Fig. 129.

It is seen that the graphs OC and OD both have a region at the lower part of the magnetization curve where they lie above the magnetization curves of the generator. Of course, the generator could not "build up" through these regions because for a given magnetizing current the condensers require more voltage than the generator can give. To make the generator build up some condenser, as H, must be used, such that its graph OH does not intersect the magnetization curve in the region shown in Fig. 129, but in using such a condenser the generator is likely to build up to abnormal values of voltage, so as soon as the machine begins to generate the condenser must be cut down to its proper value.

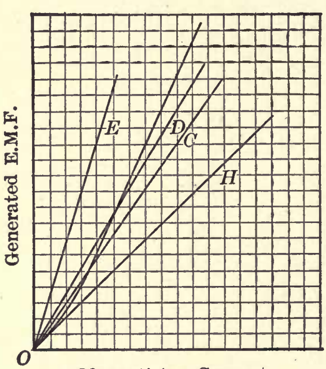


FIG. 129.

When using condenser for excitation it is not at once evident how the frequency of the generated E.M.F. is determined.

The generator will have a certain equivalent inductance L* and this, combined with the capacity C, makes a resonant circuit whose natural period of oscillation $= 2\pi\sqrt{LC}$ (neglecting the effect of resistance). Now, it may be ascertained both mathematically and experimentally that the condenser used must be such that the natural period of the system is **less than the speed of rotation of the generator.** This only emphasizes the fundamental principles noted before, that the generator slip must be negative. The speed of the field rotation with condenser excitation must be brought lower than the rotor speed, and this is done by so increasing C that the natural period of the system has some value lower than rotor speed.

The commercial application of induction generators is not very wide. Some polyphase installations for railway work utilize the generator action of an induction motor run above

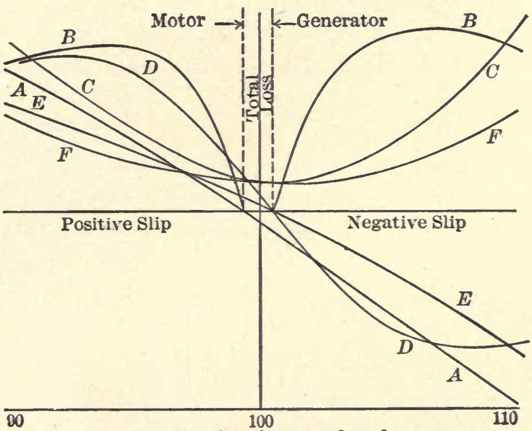


FIG. 130.

synchronous speed for a braking effect when on down grades. Of course, the power pumped back into the line by such use of the motors means just so much saved energy, as the load on the power house is decreased. Some recent installations of steam turbines, utilizing the exhaust of reciprocating engines, have used induction generators. Such generators are connected

* A more complete discussion of this point will be published in the near future. Prof. Pupin and the writer have done a lot of theoretical and experimental work on the subject and expect to soon publish an article giving the results of these investigations.

directly to the bus-bars, the necessary leading current being obtained by overexciting one of the alternators. The induction generator may be used to increase the output capacity of a station without appreciably increasing the short-circuit risk of the station.

As the power factor of induction machines is not high, the practicability of utilizing condenser excitation is not marked. The condensers would have to be exceedingly large.

Figure the size of condenser required in a 5000 K.V.A., 60 cycle (E.M.F. = 2300 volts) station of induction generators, supposing the power factor of the machines run as motors as 93 per cent and that of the load as 85 per cent.

The complete behavior of an induction generator can best be obtained from such a set of curves as is given in Fig. 130. The continuous change in the different quantities from motor to generator action is evident. For the region of the "total loss," previously referred to, the efficiency is, of course, zero. By negative power factor is meant that the magnetizing current is

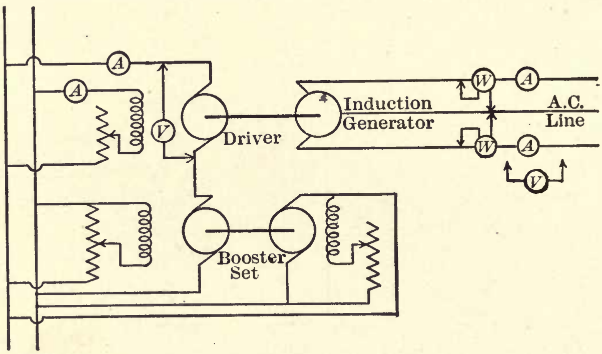


FIG. 131.

leading the terminal E.M.F. of the generator. In this set of curves the letters have the following significance.

A = torque, B = efficiency, C = total stator current, D = power factor, E = watt component of stator current, F = wattless current.

The tests to be made in this experiment are: the magnetization curve, to be obtained as previously described, by synchronous-motor excitation; external characteristic, using synchronous-motor excitation and constant rotor speed, load to be non-inductive; operating characteristics given in Fig. 130; sufficient

data to construct circle locus of current (this data is obtained exactly as though the machine being tested were a motor).

To obtain the operating characteristics as given in Fig. 130, make connections as in Fig. 131. The D.C. machine should be approximately the same capacity of the induction machine, and the booster in the D.C. armature circuit capable of boosting or crushing the D.C. line voltage by 10 per cent or 15 per cent. The A.C. generator is to be connected to an A.C. line of constant potential and frequency. In connecting the induction generator to the A.C. line care must be taken that the rotating field goes in the **same direction as the D.C. motor drives the rotor.**

Measure the core loss and friction of the driving motor at normal field current at such speeds as give to the induction generator, 90 per cent, 100 per cent and 110 per cent of rated speed. Measure D.C. armature resistance. By means of the booster vary the action of the A.C. machine over as wide a range as possible, through both motor action and generator action. Keep the field current of the driving motor constant at normal value so that the core losses may be obtained from core-loss curve previously measured. The I^2R loss in the D.C. machine can be calculated for any value of I.

Read D.C. armature current and volts, and A.C. amperes, volts and watts. Measure speed of A.C. machine either by one of the slip methods previously described in Experiment 33, or, if the slip becomes too large for this method, by a tachometer. For each load calculate the input or output of the induction machine and so obtain the set of curves called for.

Construct circle locus of current and plot on it the values of current measured in this run. Points may be plotted as described in Experiment 36.

EXPERIMENT XXXVIII.

THE SINGLE-PHASE SERIES MOTOR.

At present, nearly all electric railways in this country use the direct-current series motor as their motive power. The distribution of the electric power from the generating station to the car motor is inefficient because of the many steps involved. The power is generated as alternating current, goes through step-up transformers at the station to the high-tension transmission line and to the substation, through step-down transformers, changed to direct-current power by a rotary converter, and is then conveyed over the D.C. feeders and trolley wire to the car. Probably not more than 60 per cent of the energy generated at the main station reaches the car.

In case the alternating-current series motor is used the power distribution is much simpler. Only a step-up transformer, feeder and trolley wire and step-down transformer are used in the distributing system so that the distribution is accomplished much more efficiently, both as regards loss of power and maintenance of apparatus, than with the D.C. system. Probably if the A.C. series motor was as efficient and reliable as the D.C. series motors all railway installations would be A.C.

A D.C. series motor will run in the same direction whichever way it is connected to the line, provided the relative connection of its armature and field is undisturbed. It follows that a D.C. series motor would exert a unidirectional torque if connected to an A.C. line. Such an application of the D.C. motor is not feasible because of the high impedance of its field coils and because of the fact that its solid iron poles and yoke would not sufficiently reverse their polarity with the frequency of the A.C. supply. There would also be heavy sparking at the brushes when the motor was running.

It is evident that the field frame, as well as the armature of an A.C. series motor must be of laminated iron because of the continually reversing field flux. Because of this fact the A.C. motor cannot be as cheaply constructed nor of such rigid mechanical design as the D.C. motor.

But even when the field frame is laminated there still remain two series defects to overcome, namely low power factor and poor commutation.

The reason for the low power factor may be easily seen by considering the action of the field and armature windings. In Fig. 132 * is represented the elementary series A.C. motor. The field coil produces useful flux, which is necessary. The armature produces a vertical flux (Fig. 132), which is of no use in the operation of the motor but helps to produce a low power factor. By putting a compensating winding around the armature, containing the same number of turns as the armature, but so connected as to produce a M.M.F. opposite to that of the armature, then the armature and compensating coil will just neutralize and produce no magnetic field and so produce no lag in the current. Such a compensating winding is illustrated in Fig. 133, the compensating winding being conductively connected to the rest of the

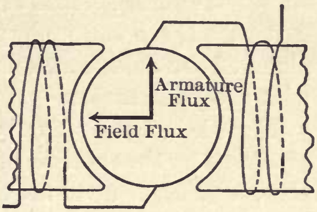


FIG. 132.

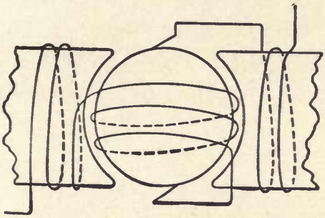


FIG. 133.

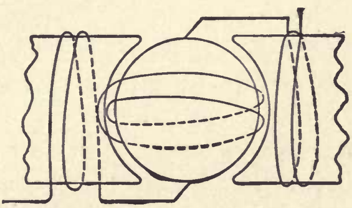


FIG. 134.

windings. As the useless armature flux is alternating, the compensating coil might simply be short-circuited as in Fig. 134, or the compensating coil may be put in series with the main field and the brushes short-circuited. In these two schemes the armature flux is neutralized by magnetic coupling of armature and compensating coil, not electrical coupling as in Fig. 133.

The next step in improving the power factor is to make the number of turns in the field as small as possible, using a correspondingly greater number on the armature. This large number

* Of course the A.C. series motor does not have actual projecting pole pieces as shown in Figs. 132–135. The field winding and field core of a series A.C. motor resemble in appearance those of an induction motor.

of armature turns does not produce a low power factor as the armature winding is compensated.

The next thing to consider is the commutation of a series A.C. motor. This is by far the most difficult point for the designer to satisfactorily solve. The armature winding of the A.C. motor is nearly the same as that of the D.C. motor so it might appear at first that if the coil undergoing commutation were in the neutral plane of the field, commutation would be no more difficult than with the D.C. motor.

There is in the A.C. series motor armature not only an E.M.F. produced by the rotation of the armature through the field flux but also an E.M.F. due to the time rate of change of the field flux, this last E.M.F. being due to transformer action of the field coils and armature coils. It will also be seen from Fig. 135 that coils *AA*, those being commutated, are in the position where the transformer action has its maximum effect, while in coils *BB* the effect is zero.

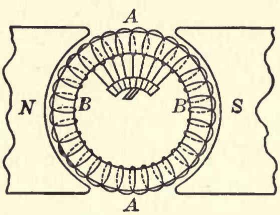


Fig. 135.

The three factors affecting this transformer E.M.F. are, the number of armature turns in series between commutator bars, the frequency of the power supply and the density at which the field is worked. Decreasing any or all of the quantities tends to improve commutation. The armature of A.C. series motors are generally made with only one or two turns between commutator bars, are operated on circuits of 25 cycles or less and the field densities employed are less than those used with D.C. motors.

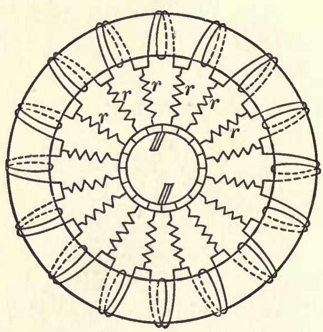


Fig. 136.

The transformer E.M.F. in the coil undergoing commutation produces in this coil a large short-circuit current, the coil being short-circuited through the brush. It is the breaking of this current as the coil moves from under the brush which produces in this type of motor the excessive sparking. One method of limiting this short-circuit current is by what are called "preventive leads," being wires of comparatively high resistance connecting the

coils to the commutator bars, as shown in Fig. 136, at r, r, etc. Suppose the preventive lead has a resistance equal to twice that of the coil itself. Then when the coil is short-circuited through the brush the short-circuit current is one-fifth as large as it would be without the resistance leads. It might seem that these leads would so increase the armature resistance that low efficiency would result. This is not the case. If there are fifty coils between adjacent brushes the armature resistance with leads is only $\frac{54}{50}$ of what it would be without the preventive leads. The resistance of the leads is large compared to the resistance of one coil, but low compared to the resistance of the whole armature.

The torque of the A.C. motor is pulsating, having a frequency twice that of the power supply; the entire field frame is subject to core losses; the short-circuit coil is subject to I^2R losses produced by the transformer E.M.F.; the power factor is always less than one; all of these effects result in less horse power per pound of material than is obtained from the D.C. series motor. Also the commutator of the A.C. motor is likely to require more attention than that of the D.C. motor.

The speed load curve of the A.C. motor is practically the same as that of the D.C. motor. The power factor of the motor increases with speed for two reasons. The reactance drop in the field winding decreases with the decrease of current at increased speed and the C.E.M.F. of rotation, which is an energy or dissipative reaction, in phase with the current, increases with the speed.

A vector diagram of the different reacting E.M.F.'s in the single-phase series motor serves well to illustrate the effect of the compensating winding, etc. In Fig. 137 the radius of the circular arc is taken equal to the E.M.F. impressed on the motor. In Fig. 137 we have

OA = Full-load field reactance drop.
AB = Full-load armature reactance drop.
OC = Full-load armature and field resistance drop.
CD = Full-load compensating winding resistance drop.
OE = Full-load current motor impedance drop, uncompensated.
EF = Full-load current motor C.E.M.F. of rotation, uncompensated.
$\cos\phi$ = Full-load current motor power factor, uncompensated.
OG = Full-load current motor impedance drop, compensated.

GH = Full-load current motor C.E.M.F. of rotation, compensated.

$\cos \phi'$ = Full-load current motor power factor, compensated.

This diagram shows that the power factor is considerably improved by the addition of the compensating winding. With compensation, of course, the only reactance drop occurring in the motor is OA, that due to the field winding. It might seem that the efficiency of the motor would be decreased with compensation owing to the extra IR drop shown at CD. The efficiency is, however, greater in the compensated motor than in the uncompensated one, because for a given output more current is required in the latter type, due to its lower power factor, and also the core loss is reduced. For a given current the speed is also considerably higher in the compensated motor; this is due to the decreased reactance drop, thereby leaving a larger percentage of the impressed E.M.F. to be used up in C.E.M.F. of rotation, and as the field is assumed the same strength in both cases the compensated motor must run at a higher speed than the uncompensated type.

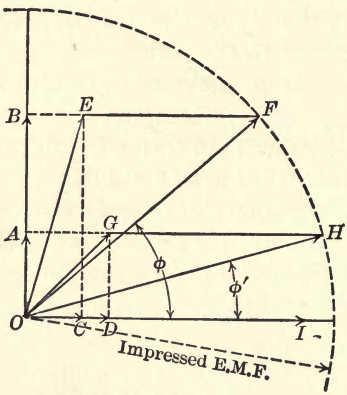


FIG. 137.

Make a test upon a series A.C. motor (using brake for load), reading volts, amperes, watts input, torque and r.p.m. Take readings in about ten steps between the load which gives 150 per cent rated speed and that which gives 150 per cent rated current, the voltage and frequency being kept constant at their rated values.

Make a similar run with the compensating winding connected in the motor circuit but reversed, so that its inductance adds to that of the armature instead of serving to neutralize it. Note the commutation in these two runs.

Make a run, having the compensating winding properly connected, using for power supply some frequency widely different from the rated value (e.g., run a 25-cycle motor on a 60-cycle line). Note and record the commutation as compared with the previous run. Make a similar run using the motor

on a D.C. line of the same voltage as the motor rating. As the losses will be less in this case than when the motor is being run on A.C. the full-load current may be reckoned as 110 per cent of the A.C. rating.

With normal frequency and various voltages (beginning with low values) and the armature locked, read watts, amperes and volts input at about six points from zero to 50 per cent or 75 per cent over-load current. Take care that the motor does not over-heat with the higher values of current. Also read amperes, watts and starting torque with full-load current, both D.C. and A.C. Calculate amperes and watts per pound-foot of starting torque for A.C. and D.C.

With current input as abscissa, draw four sets of curves, one set for each run. In each set, plot, against current input, speed, torque, watts input, current input, efficiency and power factor. From the "locked-saturation" curve and one of the readings from the first test construct the circle diagram* and predict the various characteristics of the motor.

What conclusions can be drawn from the starting torque characteristics on A.C. and D.C. power?

Note. — Care must be exercised that the motor speed does not rise to a dangerous value. A switch should be so placed that the man taking speed can immediately open the A.C. supply circuit if the speed exceeds the safe limit (5000–6000 ft./min. = maximum peripheral speed).

* For construction of this diagram see Crocker and Arendt, "Electric Motors," page 249.

EXPERIMENT XXXIX.

THE MERCURY ARC RECTIFIER.

Two electrodes in a vessel which has been exhausted to a high degree of vacuum are practically insulated from one another for ordinary voltages. The phenomena which result from a gradually increasing potential difference of the electrodes are of too involved and complex a nature to be fully analyzed here, but it is believed that this subject of conduction of electricity by gases is of enough importance to be seriously studied by the electrical engineer. It is undoubtedly true that a thorough comprehension of the novel effects produced in high tension transmission of electrical power can only be obtained by a study and application of the electron theory.*

In the analysis of the mercury arc rectifier only those fundamental facts which are necessary for an explanation of the action of the rectifier will be taken up and briefly treated. If a tube in which two electrodes are sealed is exhausted to a high degree of vacuum and an increasing voltage is impressed on the electrodes it will be found that a very small current will pass through the tube, perhaps one or two milliamperes. The gas in the tube is said to be un-ionized and in this state has a very high resistance. As the potential difference of the electrodes is increased the current will gradually rise, but until a certain critical voltage is reached will not exceed a few milliamperes.

When the potential difference reaches a value of perhaps 2500 volts the current will suddenly rise and, if the potential difference is maintained, the current will be of such a value that in a few seconds the negative electrode will melt. This sudden rise in current is explained by saying that when the potential gradient in the tube reaches a certain value the gas becomes ionized (so-called "ionization by impact") and when ionized to a considerable extent a gas becomes a fair conductor.

The potential drop is greatest near the cathode, in fact,

* The student is referred to J. J. Thomson's "Conduction of Electricity through Gases," for a complete exposition of this theory.

practically all of the E.M.F. applied to the tube is used up in overcoming the resistance from the cathode to the adjacent gas. This potential difference is called the "cathode drop," and this drop is very high when such an electrode as iron is used.

We have now the fact that an ionized gas is a conductor, but that an excessive voltage is necessary to overcome the cathode drop. As the power used in the circuit is equal to the product of volts × amperes, it is at once evident why the negative electrode gets so hot.

The next thing to consider is the action of such a tube when one electrode is made of mercury, the gas in the tube is mercury vapor, and the other electrode is some such substance as iron.

Suppose a tube made as shown in Fig. 138. The voltage of the generator may be brought to a very high value but still no appreciable current will pass through the tube because the enclosed gas is not ionized. If now, by means of the auxiliary electrode *C* and induction coil *H*, a spark is made to pass into the mercury electrode, immediately ions are freed from the mercury, the gas is ionized and a current of a value of several amperes (limited by resistance *R*) will flow from *B* to *A*. Also the potential difference between *B* and *A*, even when 10 or 20 amperes are flowing, is only about 15 volts.

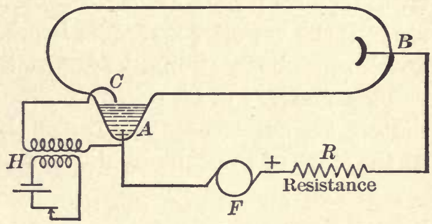


FIG. 138.

If, however, the generator terminals are reversed, making *B* the cathode, it makes no difference how well the gas is ionized, no appreciable current will flow through the tube unless an excessively high potential difference is used. We have here then the secret of the mercury arc rectifier, i.e., selective electrodes. When the mercury is used as cathode, the cathode drop is about 4 volts; when the iron is used as cathode the cathode drop is perhaps 2500 volts.

If such a tube is connected to a source of A.C. power and the gas in the tube is ionized, whenever the iron electrode is 15 volts or more higher in potential than the mercury electrode, current will flow. If the tube is connected as shown in Fig. 139 and an oscillogram is taken of the current and voltage in the circuit they would be as given in Fig. 140. Such a tube would, therefore, be

giving pulses of unidirectional current. The drop of 15 volts in the tube is nearly independent of the current flowing, so that the amplitude of the current pulses depends only on the value of the resistance R. The ionizing spark must continually be operating, otherwise current would not start to flow through the tube until excessively high value of potential difference was used. After the current starts to flow, however, the ionizing spark may be cut off and the current itself will maintain the ionization. If the current stops for a fraction of a second (something less than 0.000001 second) the vapor recovers its insulating quality and the ionizing spark must be again used to start the current flowing.

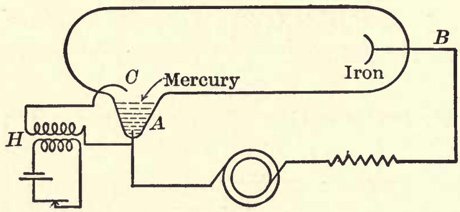


FIG. 139.

This recovery of insulating power is due to the fact that immediately the current stops flowing, the ions disappear, some going to the sides of the tube by dispersion (and clinging to the sides of the tube) and the rest recombining. As the recombination is not very rapid at low pressure it is probable that dispersion accounts for most of the effect.

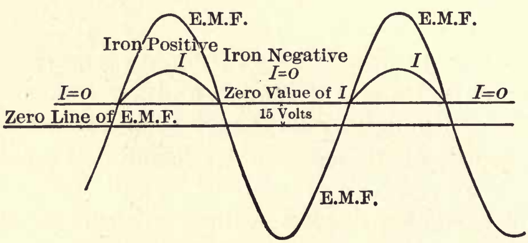


FIG. 140.

The next step in the development of the rectifier is to so construct it that both alternations may pass through the tube, both going through the tube in the same direction. This is done by constructing a tube with one mercury electrode and two iron electrodes, as shown in Fig. 141. The tube is connected to the line as shown. The two sides of the A.C. line are connected to the two anodes B and D, and the cathode is connected to the junction of the two reactance coils F and G. Between A and E

are connected some storage cells which may be charged from the tube.

The operation is now as follows: the gas being supposed ionized continually (by electrode C and spark if necessary). When B is positive with respect to A (and more than 15 volts above A in potential) current will flow from B to A, through the battery K, through F and so back to the A.C. line. During the next alternation D becomes positive with respect to A, so current flows from D to A, through K and G and so back to the line. Both pulses of current pass **through K in the same direction,** so that this tube will give a unidirectional pulsating current as shown in Fig. 142. As the ionization, produced by the current itself, would disappear once every alternation (at the points of zero current in Fig. 142), the gas must be kept continually ionized by the coil H and electrode C. As this is inconvenient, a method is sought to overcome this difficulty.

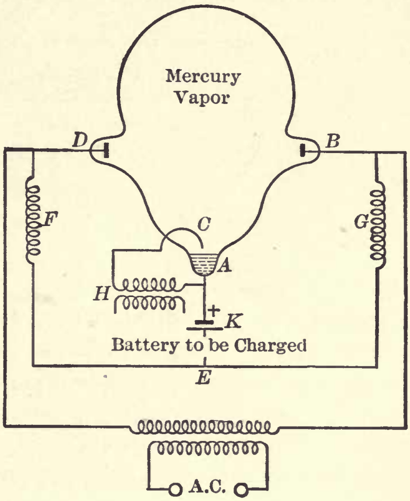


FIG. 141.

So long as current is flowing into the cathode the gas remains ionized and it matters not from which anode the current is coming. If, therefore, the current can be made to start from D before the current from B has reached zero value, the tube will keep itself continually ionized and the electrode C need be used only for starting.

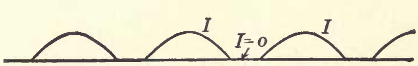


FIG. 142.

This is the condition which actually obtains in the tube and it is brought about by the reactance coils F and G. In Fig. 143 is drawn the wave of potential difference between B and A. As soon as a value of (15 volts + C.E.M.F. of battery) is reached current begins to flow at a, Fig. 143, and follows practically sine wave as shown by the dashed line. If the reactance coil did

not come into play the current wave would continue as shown by the dashed line and would reach zero value at time *b*. The current from anode *D* will not begin until time *c*, so that were it

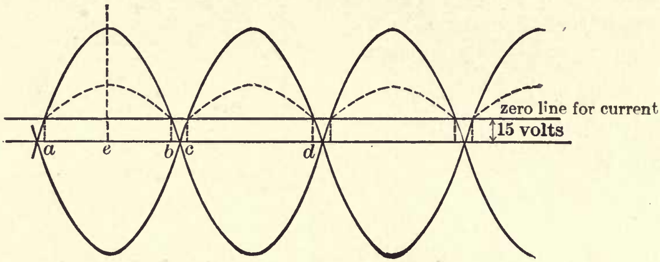


FIG. 143.

not for the reactance coils during the time (*b–c*) there would be no current in the tube and the gas would lose its ionization.

During the time (*a–e*) the reactance coil *G* is storing magnetic energy and as the current starts to decrease at time *e*, the reactance coil begins to discharge its energy and tends to sustain the current. This results in a change in the current wave form as shown in Fig. 144, where the E.M.F. waves and current waves

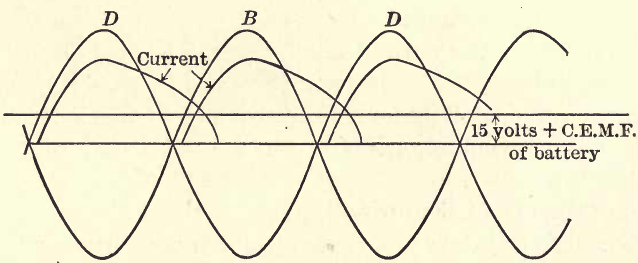


FIG. 144.

of both anodes are shown. The two current waves now lap over one another by several degrees so that the current wave through the cathode is shown as in Fig. 145. Hence, a tube constructed with two anodes and used with reactance coils will give through the cathode a pulsating unidirectional continuous current. The variation in amplitude may be 30 per cent or 40 per cent of the maximum value.

As a storage cell becomes charged its C.E.M.F. rises, and, therefore, arrangement must be made whereby the rectifier voltage may be increased as the charge continues. Also different

numbers of cells will be charged at different times, so that here also different voltages are desired. The rectifier as actually

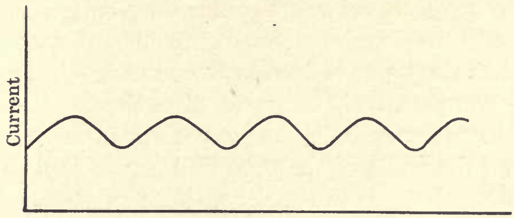


FIG. 145.

installed permits an adjustment of the voltage before starting the charge and then a variation in the voltage as the charge continues. A variable reactance is placed in the A.C. supply line, which reactance is varied with time of charging. In place of the transformer and reactance coils an adjustable auto-transformer is used, as shown in Fig. 146. In this figure the starting arrangement is also shown. Instead of using a spark from an induction coil to start the ionization, an arc is used. With switch S_1 closed the tube is tipped so that mercury connects the two electrodes A and C, and a current flows, the amplitude being limited by resistance R_1. When the tube is tipped back an arc is formed as the bridge of mercury is broken, the gas is ionized and if switch S_2, connecting in the starting resistance R_2, is closed, the tube will begin to operate, using R_2 as load. When the tube has been running a few seconds through R_2, to get warmed up, S_3 may be closed and S_2 opened. S_1 should be opened as soon as the tube is operating.

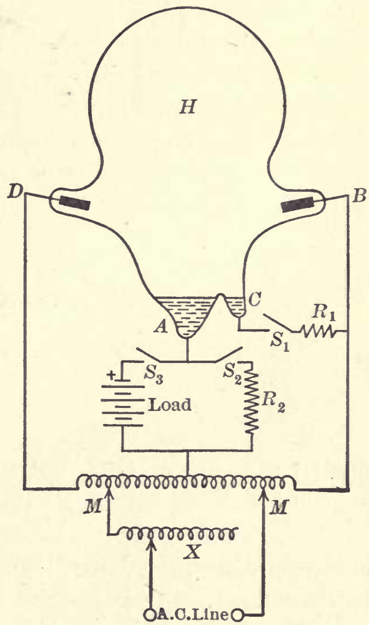


FIG. 146.

The D.C. voltage available at the load is about 25 per cent of the A.C. voltage between the two anodes D and B. In starting

to charge, the contacts *MM* of the autotransformer are set at the proper tap with the reactance coil *X*, all in the circuit. Then as the charge progresses the reactance may be gradually cut out if it is desired to maintain the charging current constant.

As considerable power is used in the tube itself, the upper part *H* is made comparatively large to give the required radiating surface. The mercury boils and vaporizes from the cathode, is condensed on the walls of the tube and serves well to distribute the heat uniformly over the surface of the tube.

The efficiency of an arc rectifier is nearly independent of load (the efficiency of the tube itself being strictly so), but depends

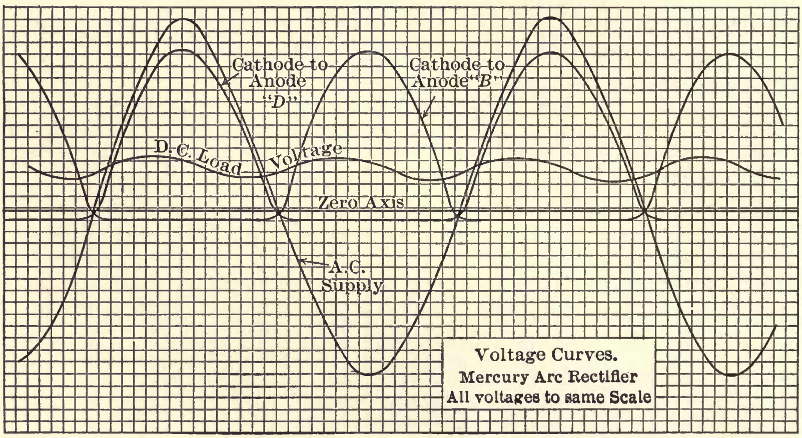


Fig. 147.

upon the value of D.C. voltage at the load, the efficiency increasing with the voltage.

The power factor of the rectifier depends somewhat upon the load and also upon how much of the reactance coil *X* is being used, but generally it is well up towards 85 per cent or 90 per cent.

The rectifier has an inherent regulation of about 20 per cent from rated load of the tube to the minimum load the tube will carry.

The wave forms of the current and E.M.F.'s in the different parts of the rectifier circuit are to be obtained either by ondograph or oscillograph. The waves to be taken are A.C. line voltage and current; voltage between each anode and cathode;

current furnished by each anode; D.C. current (cathode current), and voltage across load. All of these curves must be taken with the same setting of the curve-tracing device so that their proper phase relations are obtained; sample curves from a 10-ampere 220-volt tube are shown in Fig. 147 and Fig. 148; they were taken by means of the ondograph.

Now insert a high inductance in the load line (a transformer coil, e.g.) and take another curve of D.C. current to see if pulsations are reduced.

Make runs to obtain efficiency, power factor and regulation as follows:

Reactance X all on, readings of A.C. line volts, current and watts, D.C. current and volts, for $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, full and $1\frac{1}{4}$ rated

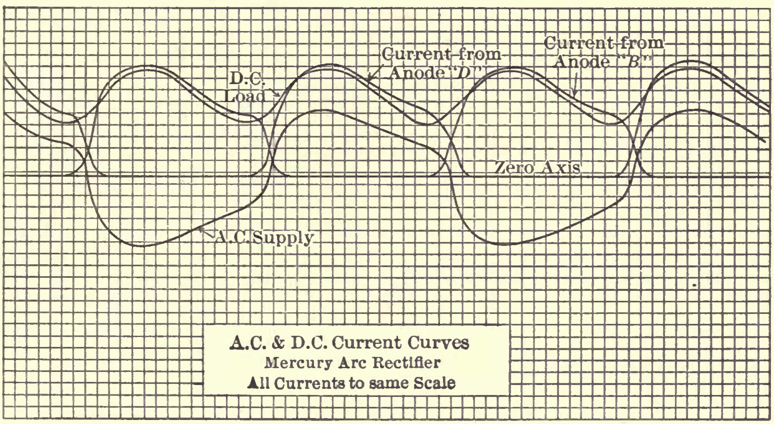


FIG. 148.

current. Run to be made with autotransformer set to give highest D.C. volts.

Similar run with autotransformer set to give minimum D.C. volts.

Take similar runs with reactance all cut out.

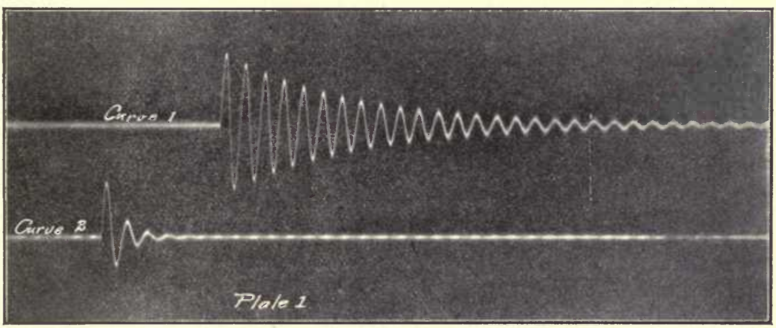
For all runs plot efficiency, power factor and D.C. volts against D.C. load current as abscissa.

Note. — In measuring a pulsating unidirectional current or voltage, A.C. instruments should be used, taking reversed readings to get rid of error caused by stray field. D.C. instruments will not give accurate indications when the pulsations have an appreciable magnitude. Why?

APPENDIX.

ILLUSTRATIONS OF THE USE OF THE ONDOGRAPH AND OSCILLOGRAPH FOR SOLVING SOME OF THE MORE INVOLVED QUESTIONS ENCOUNTERED IN A COURSE ON ALTERNATING-CURRENT TESTING.

An inductance of .11 henry having a resistance of 1.4 ohms was connected through a resistance of .6 ohm to a condenser of 80 microfarads having an inappreciable series resistance. A switch was placed in the circuit so that the connection diagram was as shown in Fig. 18 of the text. The condenser was charged to 100 volts difference of potential and then the switch was closed, allowing the condenser to discharge through the inductance and resistance. The oscillatory current in the circuit is shown by



curve 1, Plate 1. The frequency of the discharge by the formula, $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$, (resistance neglected) was calculated to be 53.5 cycles per second while the film measured about 54 cycles per second.

The resistance of the circuit was then increased to about 8 ohms and, when the condenser was again charged to 100 volts, and discharged through the inductance, curve 2, Plate 1, was obtained. The very rapid damping, as compared to curve 1, is at once seen. The change in the period of the current due to the increased resistance is not measurable on the film. The resistance of the circuit

was then increased to about 150 ohms and curve 1, Plate 2, was obtained, to the same scale as the curves of Plate 1. Upon changing

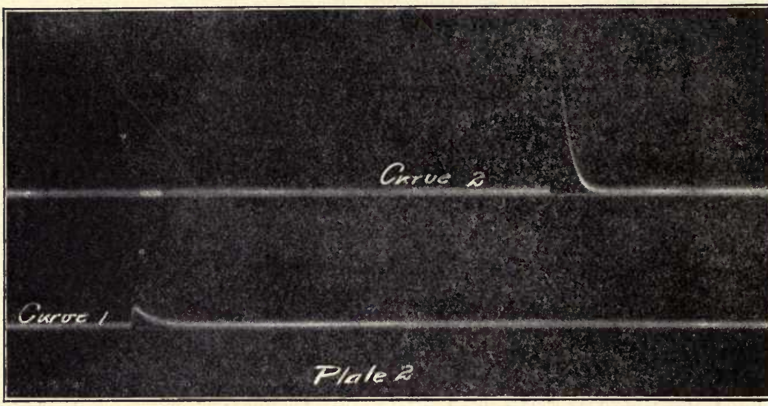


Plate 2

the resistance of the oscillograph circuit curve 2, Plate 2, was obtained as the form of the discharge of the condenser. Evidently there was no oscillatory character to the discharge current in this circuit. Upon calculation it will be seen that with 150 ohms in the circuit the term $\frac{R^2}{4L^2}$ is greater than the term $\frac{1}{LC}$ and, by inspection of the equation on page 38 of the text, it is evident that no oscillation will take place under such a condition.

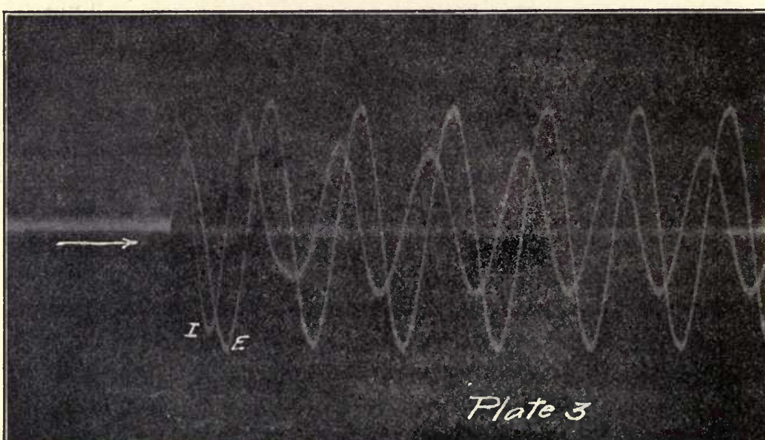
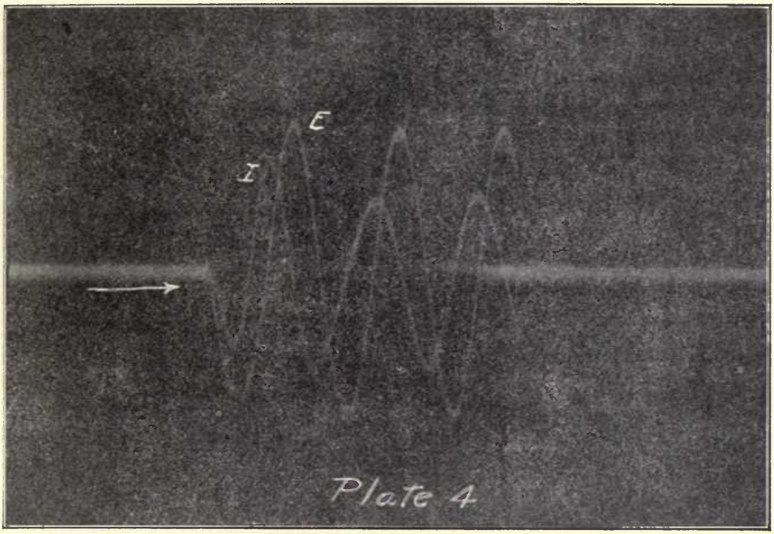


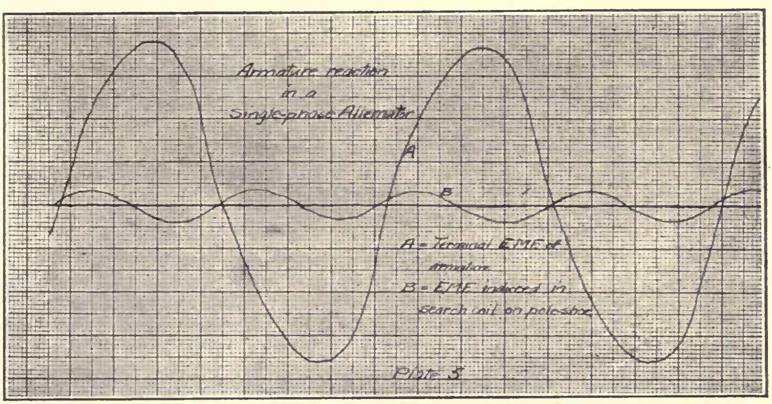
Plate 3

An inductance of about .005 henry was placed in series with a resistance of about 10 ohms and a condenser of 80 microfarads and

was switched to a 110-volt 60-cycle line. The peculiar shape of the starting current is seen by reference to Plates 3 and 4. Of

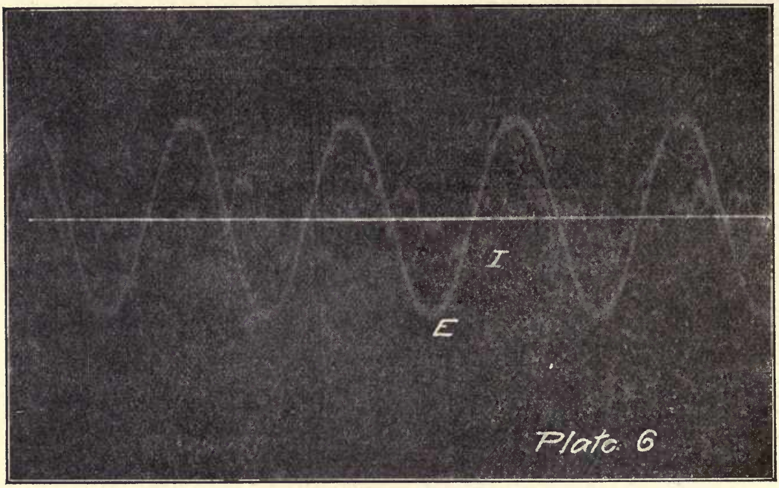


course in the circuit used the damping factor was very large, as the inductance was of such a small value. The current reached its steady state in this circuit in about two cycles.



The curves of Plate 5, which were taken by the ondograph, show the effect of the armature reaction upon the field of a single phase alternator. The curve (A) shows the voltage induced in the series field of the machine and curve (B) gives the armature E.M.F. It is

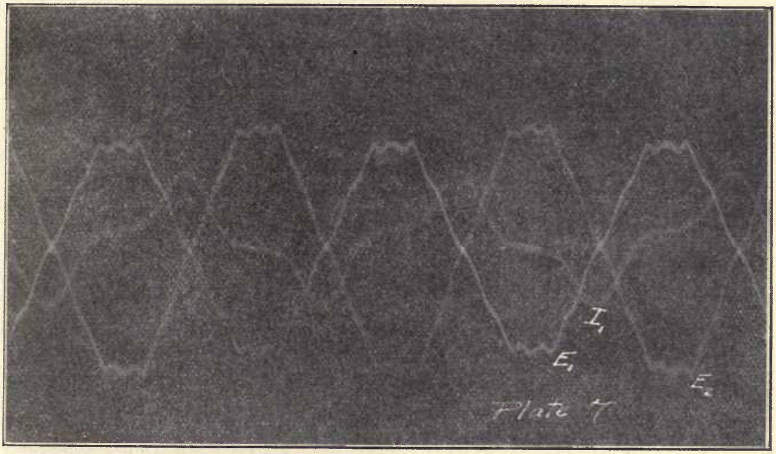
seen that curve (A) has twice the frequency of curve (B). The machine used was designed as a rotary converter; hence, the presence of the series-field winding. It was used instead of winding an additional search coil on the pole face. When the machine was loaded two phase there was no perceptible E.M.F. induced in the series-field winding, showing that the balanced polyphase load produced no pulsations in the field strength.



Two alternators of different design were being operated in parallel. It was impossible to reduce the current circulating between the two machines to less than 3 amperes, which was about 15 per cent of full-load current. A low resistance was put in the circuit of the two armatures and an oscillograph record of this circulating current was taken and it is reproduced in Plate 6. The line E.M.F. is given as reference curve. It may be seen that this circulating current is very complex in form, so that it could not be eliminated by variation of field strength, etc. The shape of this circulating current is probably due to the fact that the two alternators used had a different number of armature teeth per pole. Other machines tested gave even more complex curves for circulating current.

The question, as to whether or not the secondary E.M.F. of a transformer is exactly similar in form to the impressed E.M.F., was raised by one of the students. It seemed possible that the effect of the losses in the transformer core might be such that slight

irregularities in the impressed E.M.F. might be eliminated in the secondary E.M.F. So various forms of wave forms were impressed upon a transformer having quite high core losses but the results showed that, within the limits of accuracy of the curve-tracing



apparatus, the two wave forms were exactly similar. Plate 7 shows the result of one test; the impressed E.M.F. is shown at E_1, the primary current at I_1, and the secondary E.M.F. is shown by the curve E_2.

The alternator used in this test is the one referred to in the discussion of Fig. 87 of the text. A very pronounced eleventh harmonic is present in the wave form.

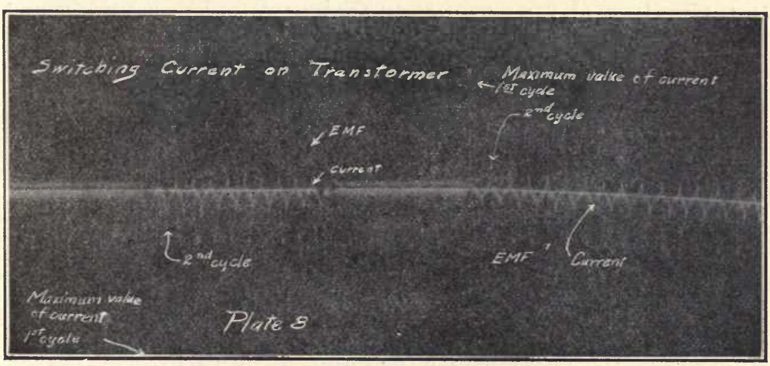


Plate 8. The excessive current taken by a transformer on being switched to a line of normal voltage is well shown on this oscillo-

gram. It is seen that the large value of current lasts for only a very few cycles and that the magnitude of the current at starting may vary greatly. On Plate 8 the magnitude of current, when the switch was closed the first time, is about ten times normal magnetizing current and the next time the switch was closed the first rush of current gives only about six times normal current.

It is quite likely that if the times of disconnecting the transformer and of switching it on the line again should happen to be the worst possible, as described in the text in connection with Figs. 59 and 60, that the starting current would be several times the full-load current of the transformer.

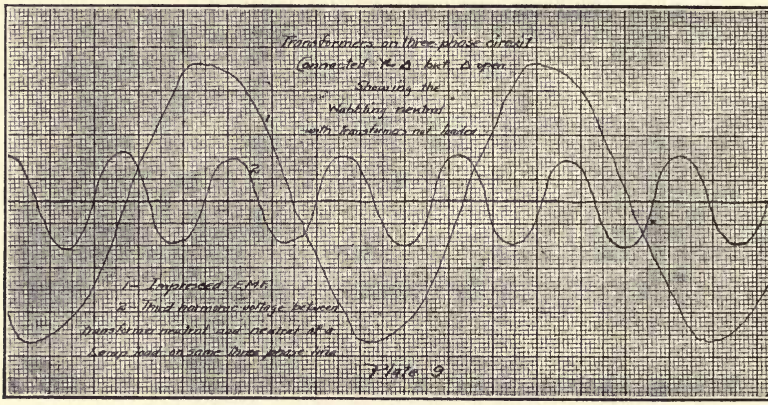
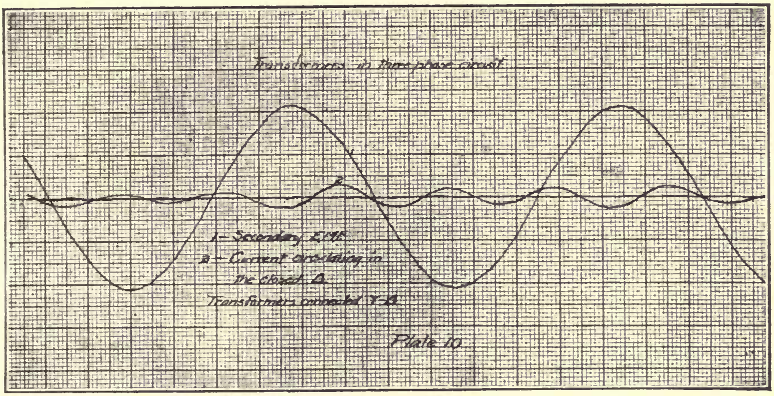


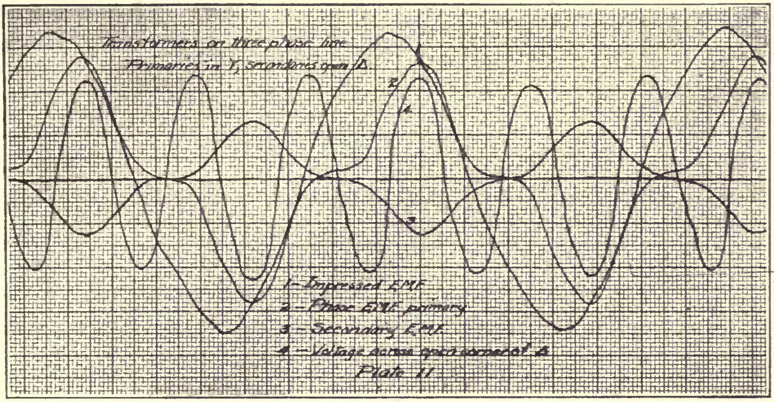
Plate 9. The idea of the "wabbling" neutral is well brought out in this ondograph record. The three transformers were connected $Y - \Delta$, and the delta was left open. The ondograph traced the curve of voltage between the center of the Y (the "wabbling" neutral) and an artificial neutral obtained by connecting to the line feeding the transformers three noninductive resistances connected in Y. The line E.M.F. wave is drawn for reference. The magnitude of the third harmonic was about 18 per cent that of the line E.M.F.

The open delta of Plate 9 was closed through a low resistance and the "wabbling" neutral became stationary, the ondograph showing no difference between the transformer neutral and the artificial neutral. There was a third harmonic current, however, flowing around the closed delta and this is shown on Plate 10, as is the secondary E.M.F. wave for reference. The current circulating

in the delta was evidently made up of some higher harmonics besides the third, as its amplitude varies in a series of "beats."



The set of curves given in Plate 11 shows the voltage relations in three transformers connected $Y - \Delta$, the delta being open and the secondaries unloaded. The line voltage (1) is nearly a sine wave; the form of the voltage wave across one primary coil (2) is very



much distorted, there being present a third harmonic of considerable magnitude; the secondary voltage wave (3) is exactly similar to that of the primary, and across the open point of the delta there is a large voltage consisting for the most part of third harmonic (4). This curve (4) is not entirely made up of the third harmonic, as may be seen from the variation in its amplitude. The voltage

across the open delta as recorded by a voltmeter was more than half as large as the normal secondary voltage. In this case the secondary voltage was 110 and the voltmeter across the open delta registered 72 volts. When the delta was closed, however, the current circulating around the closed delta was less than one ampere.

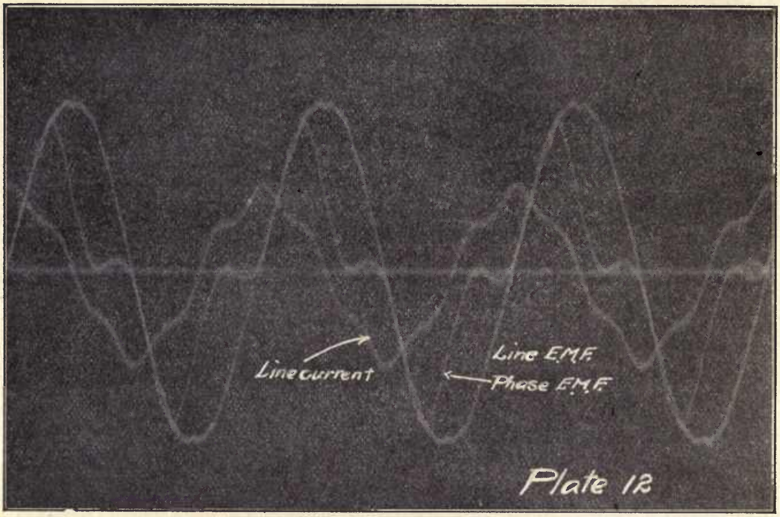


Plate 12

With the transformers connected $Y - \Delta$ and open delta, no load, an oscillogram (Plate 12) was taken of line voltage, primary phase voltage, and line current. The two voltage curves are to the same scale, the primary phase voltage consists nearly altogether of fundamental and third harmonic. The line current has a complex shape, there being present very large third and fifth harmonics besides the fundamental.

The secondary delta was closed, but with no load; the quantities of Plate 12 were photographed as shown in Plate 13. The phase voltage is, as nearly as can be detected, a pure sine wave (with exception of the fluctuations same as in line E.M.F., due to teeth of generator) and in the line current there is no third harmonic; the fifth harmonic, however, seems to be as large as in Plate 12.

The transformers were connected $\Delta - Y$ and not loaded; the primary phase current, line current and primary E.M.F. were obtained as given in Plate 14. It is seen that in such a connection the primary current has the characteristic form of a magnetizing current; in the line current there is again a very pronounced fifth

harmonic. The curves described in Plates 9 to 14 are for unloaded transformers; when load is put on the transformers these irregularities nearly disappear.

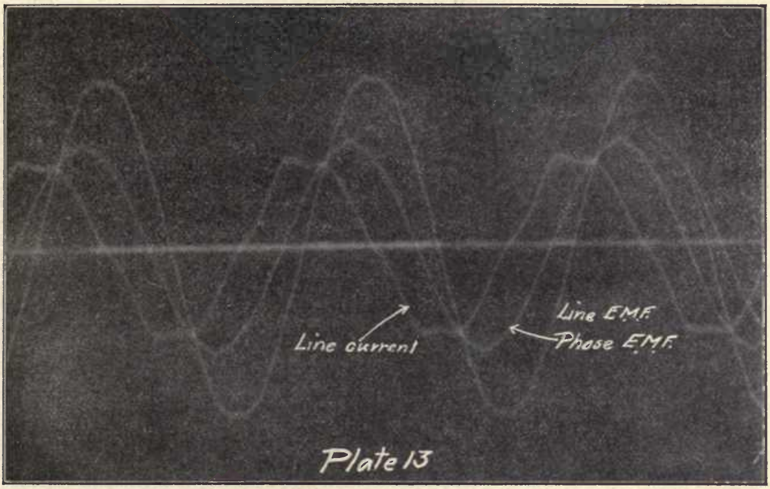


Plate 13

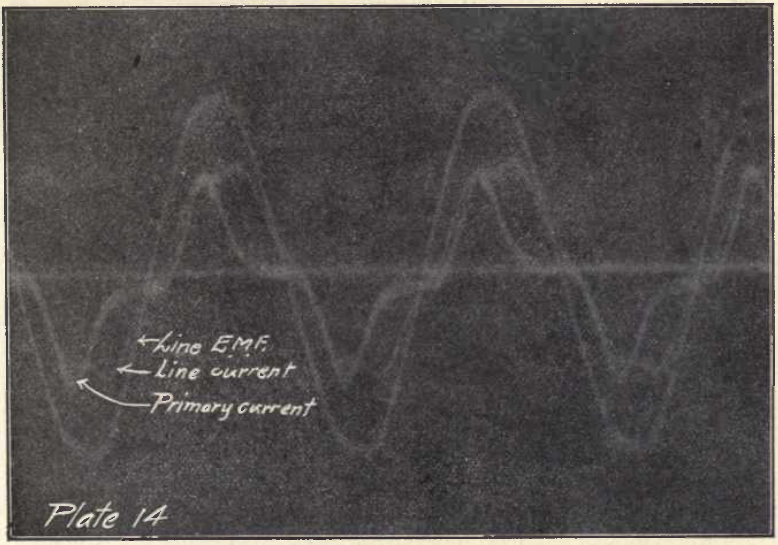
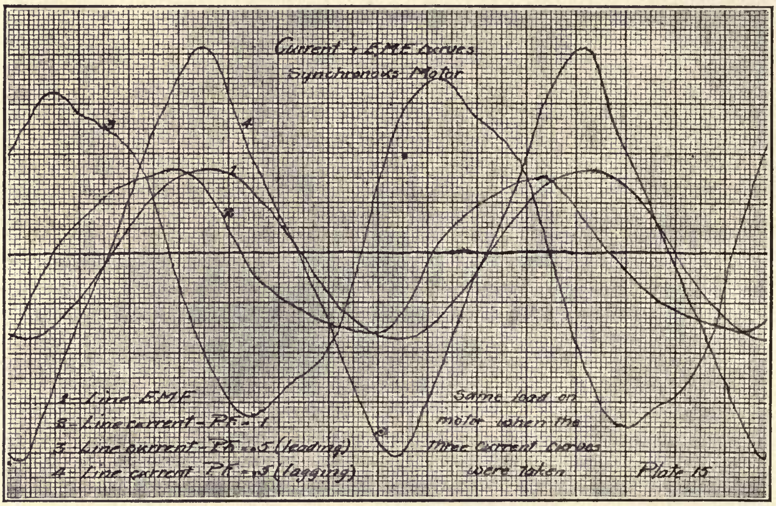


Plate 14

It has been shown in the text that in a polyphase synchronous machine there exists a nonpulsating armature reaction which will tend to concentrate the field flux in one tip of the pole or the other,

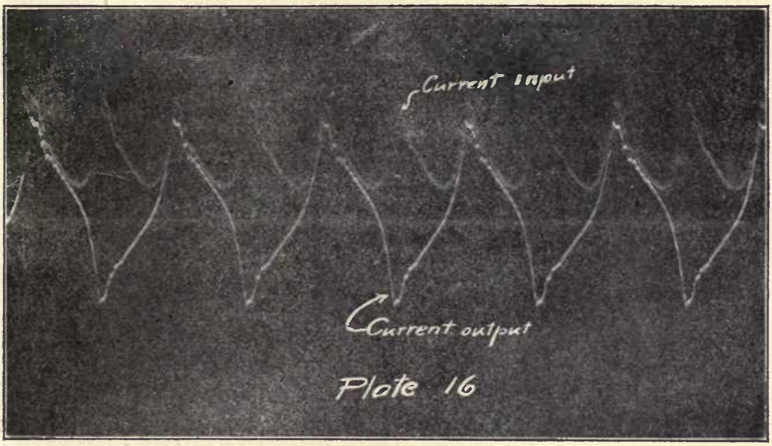
according as the motor is under- or over-excited. It is natural to suppose that the armature current under such conditions may not be a true sine wave even though the impressed E.M.F. be a pure sine wave. Some current curves were obtained by the ondograph and they showed that, on the machine tested, a very noticeable distortion of the current form took place as the field strength was varied through a very wide range. Curve 2 of Plate 15 shows the current taken by the motor when about $\frac{1}{4}$ load was being carried at power factor equal to one; even at this power factor the current



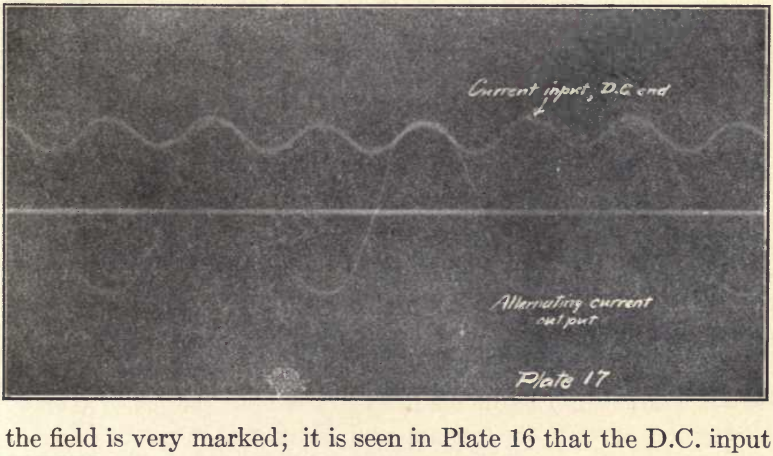
form was distorted, probably due to the fact that the armature reaction (the machine was a three-phase one) had crowded the field flux into one tip of the field poles. Curve 1 gives the form of the line E.M.F. Curves 3 and 4 show the form of current waves when the machine was carrying the same load as before, but the field was first over-excited and then under-excited. The power factor for both these curves was approximately .5. It will be noticed that a peculiar distortion takes place in these two curves; apparently there is a large third harmonic in each wave but the phase of this third, with respect to the fundamental, is different in the two cases.

In a single phase rotary the armature reaction is pulsating and therefore the strength of the magnetic field must pulsate. This being the case one would expect that if the rotary was run inverted the counter E.M.F. generated by the armature would not be a con-

stant quantity and that the current input, therefore, would not be uniform even though the impressed D.C. voltage were constant. So oscillograph curves were taken, showing the current input and



current output of a small single-phase rotary. On one machine tested, (a Crocker-Wheeler, bipolar, ring wound, 32 coil armature) the field was not stiff and the armature magneto-motive force was comparatively large, so that the effect of armature reaction upon



the field is very marked; it is seen in Plate 16 that the D.C. input pulsates almost 200 per cent and that the current taken from the A.C. end is very far from being a sine wave. It was thought at first that perhaps the angular velocity of the armature was variable, so a heavy flywheel was mounted on the armature shaft, but

the shape of the two curves was not altered by this addition. In Plate 17 are given corresponding curves for a 7 K.W. Westinghouse 4 pole rotary, which had a low armature m.m.f. and a comparatively stiff field. It is seen that the pulsations in the D.C. input are very much less marked than with the other rotary.

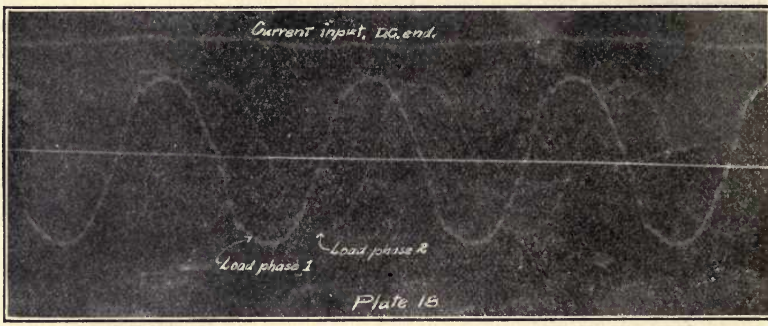


Plate 18

In the polyphase rotary the armature reaction (when there is one) is constant and not pulsating. We should, therefore, expect that in a polyphase rotary, running inverted and loaded equally on the different phases, the C.E.M.F., and hence the current input, would be constant. Plate 18 gives a set of oscillograph curves to show that this deduction is borne out by results actually obtained from such a machine.

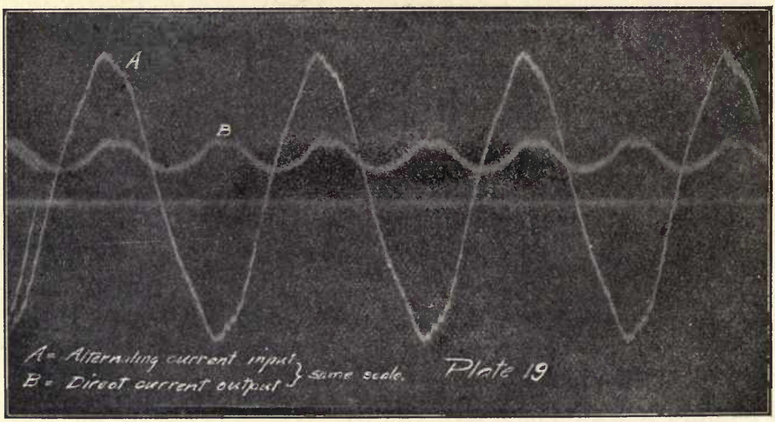
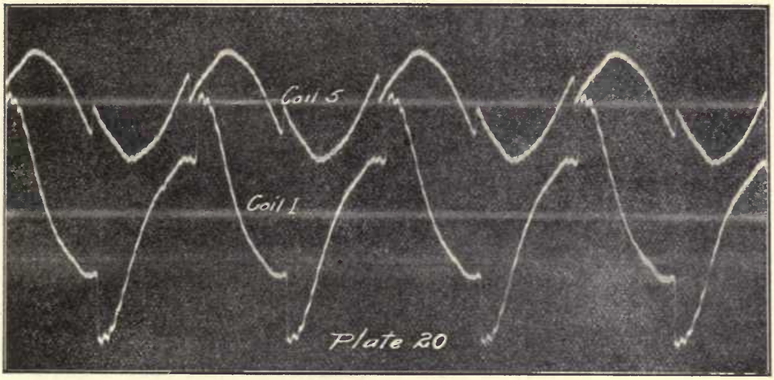


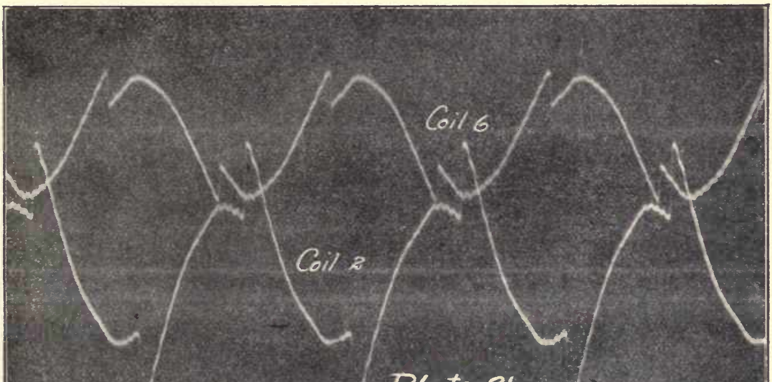
Plate 19

The rotary used in obtaining the curves of Plate 16 was run from the A.C. end, and the curves given in Plate 19 were obtained. It is seen that the effect of armature reaction does not affect the

form of the A.C. current input to a very marked extent but that the D.C. voltage, and hence the current output, pulsate with double frequency. The machine used for Plate 19 was the same as used for obtaining Plates 20–23.

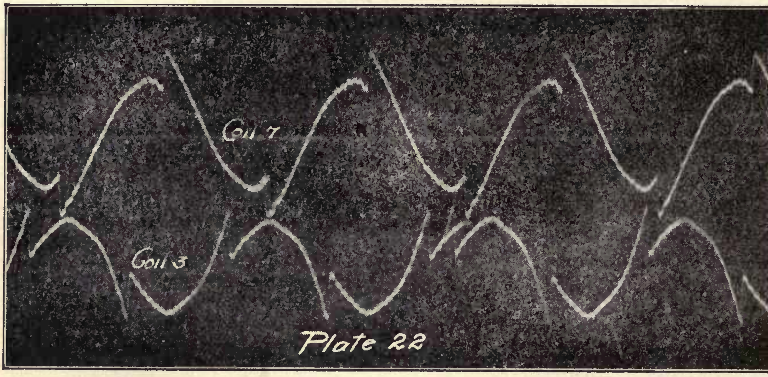


The question of the current forms in the different coils of a rotary converter is easily solved analytically or graphically, but it adds somewhat to the student's confidence in his results if the curves are actually obtained by some curve-tracing apparatus from a

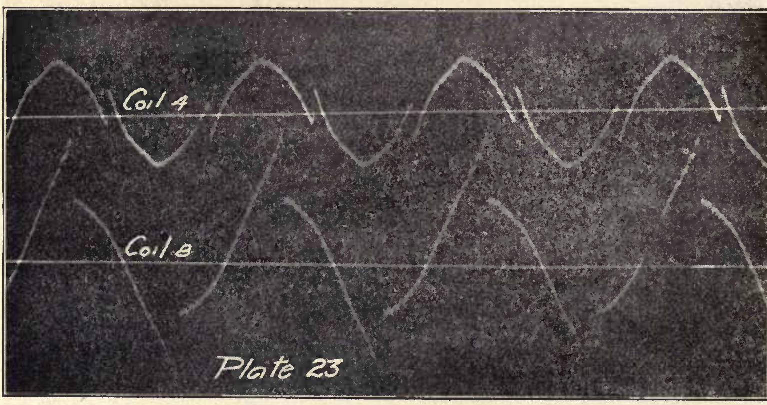


machine in operation. So with this purpose in mind we fitted up a small single-phase rotary. The armature consisted of 32 coils and the field was bipolar. Every other coil of the armature winding was opened (on the end opposite the commutator) and a small amount of resistance, wound on a bobbin, was inserted in the coil

and mounted on the end plate of the armature. An extra pair of small slip rings was mounted on the rotary shaft and these rings could be connected across any one of the inserted resistance spools. If then the oscillograph was used to show the form of the

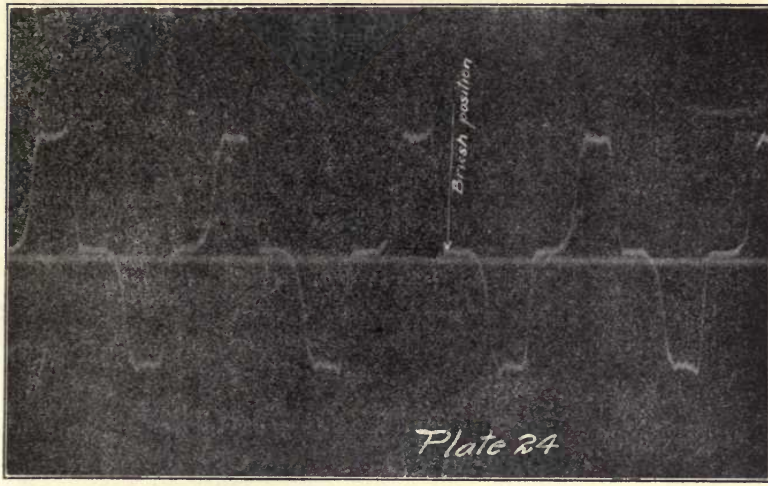


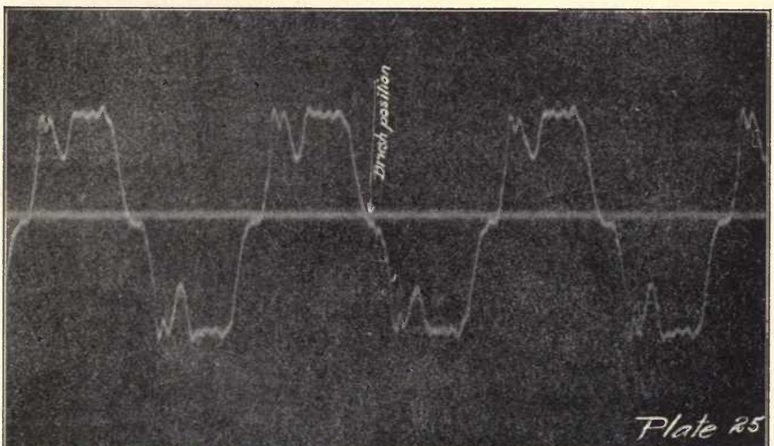
IR drop across these different spools the oscillogram would really show the current form in that coil in which the spool in question was connected. The coils in which the resistance spools were inserted were numbered consecutively, 1 being in a coil to which one of the



A.C. taps of the rotary was connected. Then coil 2 is really the third coil from the A.C. taps; coil 3 is really the fifth coil, etc., so that coil 8 is the coil nearly opposite to that in which resistance 1 is inserted. Evidently coils 9–16 would give results similar to those obtained from 1–8 and so the curves are not given here.

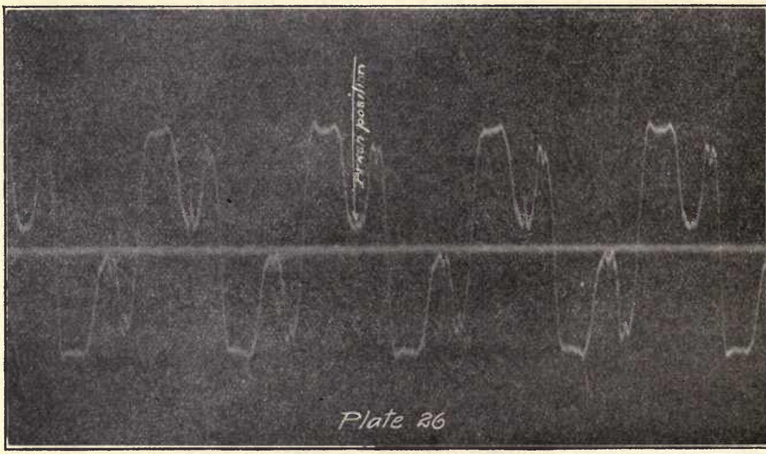
In Plates 19–22 are given the current forms of every other coil, from one A.C. tap to the next A.C. tap. In spite of the fact that the D.C. current output is not uniform (see Plate 19) and the low efficiency of the rotary used, the curves obtained are strikingly similar to the curves obtained theoretically.





The field forms of the auxiliary pole rotary were examined by taking oscillograms from a search coil made up of 10 turns of fine wire laced into two slots spaced 180° apart. This coil was connected to a couple of extra slip rings mounted on the shaft of the

rotary. Plate 24 gives the field form when the main pole only was excited; Plate 25 that produced by the main pole and auxiliary pole both carrying the same current and both poles, main and auxiliary, magnetized in the same direction; that is, the main pole and its adjacent auxiliary have the same polarity. Plate 26 shows the field form when the two poles had the same current strength as in Plate 25, but the current through the auxiliary pole had been reversed so that its polarity was opposite to that of its adjacent main pole. It will be noticed that although the auxiliary pole carried the same current when Plates 25 and 26 were taken, the

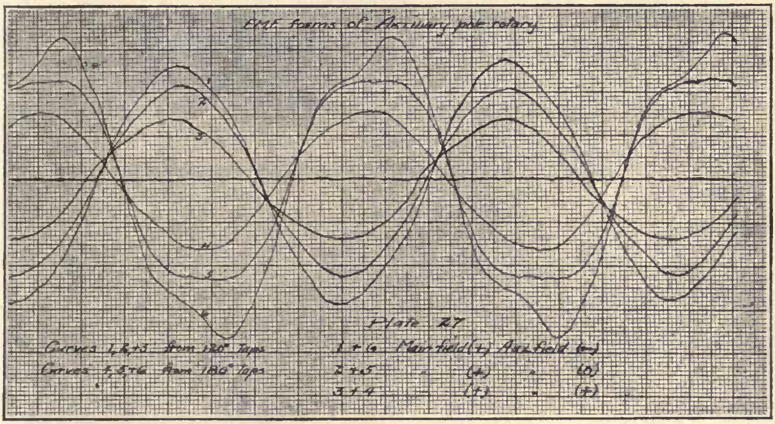


strength of magnetic field under the auxiliary pole in Plate 26 is less than that shown in Plate 25. This is due, of course, to the much greater leakage flux when the main and auxiliary poles have opposite polarity than when they have the same polarity.

The zero lines of the three curve sheets are slightly displaced but it will be noticed that with the fields excited with opposite polarity, as in Plate 26, there is considerable flux at the point of commutation and sparking occurred while the curves were being taken. The main field should have been reduced to overcome this difficulty.

Plate 27. The forms of the E.M.F. waves generated between the different taps of a regulating (auxiliary) pole rotary were examined by the ondograph for various conditions of field excitation. Curves 1, 2 and 3 represent the wave form of voltage generated between 120° taps; the machine was being run as an inverted rotary, un-

loaded, when these curves were taken. Curves 4, 5 and 6 are corresponding curves taken between 180° taps. Curves 2 and 5 were taken with main field only excited. Curves 3 and 4 were taken with the auxiliary pole excited in the same direction as the main field, while curves 1 and 6 show the results when the auxiliary pole had polarity opposite to that of the adjacent main pole. It will be noticed that the E.M.F. generated between the 180° taps suffers



quite a large distortion as the field distribution is altered but that the voltage between 120° taps retained, under all three conditions, nearly a sinusoidal wave form.

There is much material for analysis in this plate of wave forms but it is not thought well to introduce it at this point. It will be remarked, however, that a field giving a wave form as shown in curve 6, would be subject to very great changes if the rotary were run direct (i.e., A.C. to D.C.) and the impressed E.M.F. was a sine wave.